Solvency, Capital Allocation and Fair Rate of Return in Insurance*

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Introduction

Regulatory solvency requirements an important determinant of insurer (and bank) capital with product pricing and rate of return implications.

Economic capital impacts on pricing in multiline insurer (multiproduct bank) and is based on capital allocation to line of business and return on capital (RAROC).

Methods for fair rate of return requirements for regulated lines of insurance business often involve allocation of capital to the line and a return on equity by line.

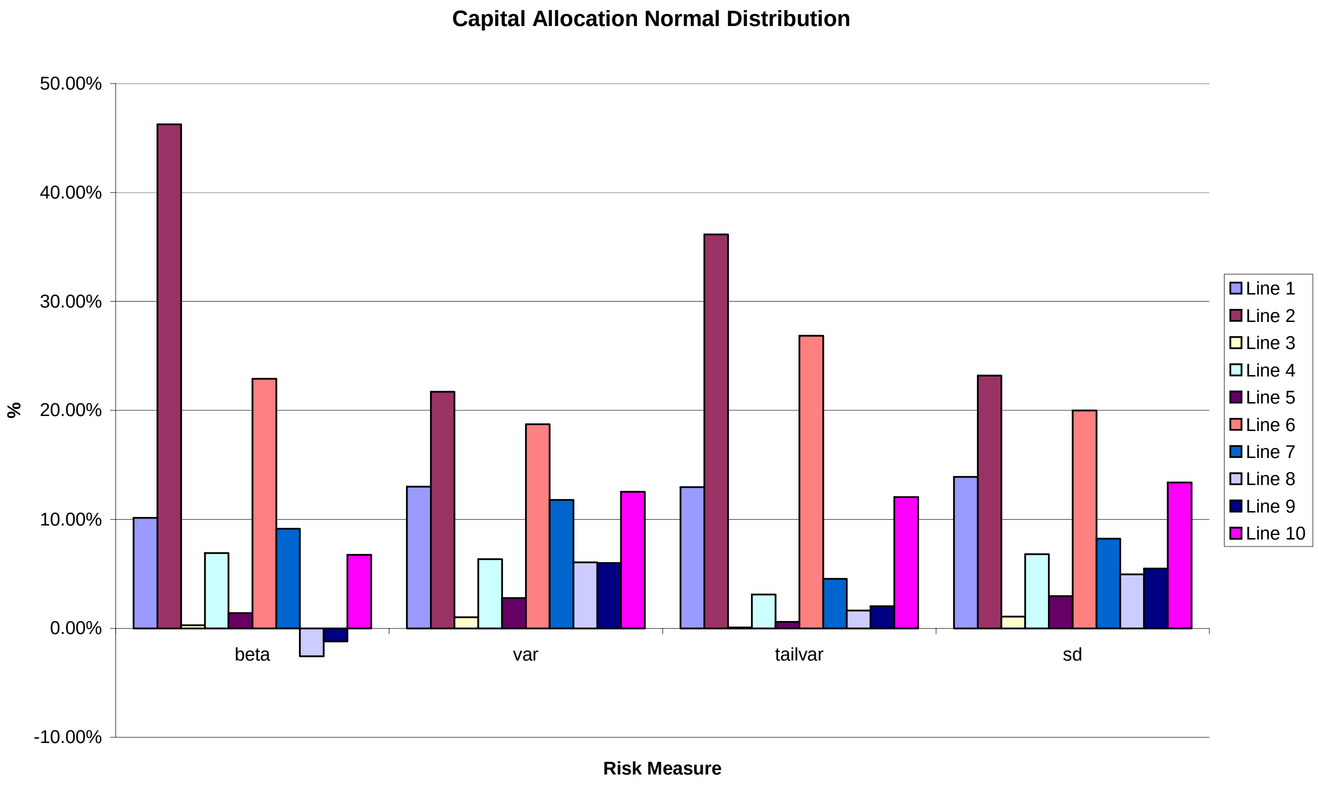
Economic capital approach allocates capital to lines of business using a risk measure and capital adds across lines (allowing for diversification) to determine enterprise wide capital.

Issues

- Many different risk measures - VaR, ruin probability, TailVaR, Expected Policyholder Deficit, Default Put Option. Which measure makes most economic sense?

- Many different approaches to allocating capital to line of business - proportional to risk measure, proportional to liabilities, marginal allocations (Merton and Perold and Myers and Read), equal expected returns to capital, covariance of losses. How to determine an economically sensible measure?

- Capital is available to support all lines of business. How to allow for this in allocating capital to line of business?



Capital Allocation with Panjer data and different Risk Measures

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Research summary

Allocation of capital is determined using a complete markets arbitrage-free model where the allocated capital "adds up" and is consistent with the fair (or economic value) of the balance sheet assets and liabilities.

Pricing by line of business does not require full capital allocation but does require the allocation of the insurer default option (insolvency put).

Capital can be allocated to provide any given expected return on capital by line through an internal (notional) allocation of assets. Capital allocation is "irrelevant" for many financial decisions.

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Research summary

The insurer default option value is an important risk measure - similar to credit spread on corporate debt that reflects probability of insolvency, the expected severity of the insolvency and based on risk neutral probabilities (risk adjusted).

Derivation of closed form results for complete markets, arbitrage-free allocation of insurer default option value to lines of business for an insurer balance sheet for fair pricing.

Economic Balance Sheet

- Determined by investment strategy of the company given by w_j the weight of asset $j = 1, \ldots, J$ in the insurer's portfolio and the end of period payoff distribution for the assets $j = 1, \ldots, J$

$$A = V_A (1 + R_A) = V_A \left(1 + \sum_{j=1}^{J} w_j R_{A_j} \right)$$

- Value of the assets

$$V_A = \sum_{j=1}^{J} E^Q \left[\frac{A_j}{1+r} \right] = E^Q \left[\frac{A}{1+r} \right]$$

where Q is the risk neutral pricing measure.

Economic Balance Sheet

- Insurer writes multiple (K) lines of business denoted by $k = 1, \ldots, K$, could be individual policies

- Line of business k incurs the random claim amount L_k at the end of the period, assuming unlimited liability. L_k is not affected by the amount of capital, dividend policy, investment policy, reinsurance strategy and any other actions of the insurer that may impact on its ability to pay the liabilities under the insurance contracts.

Economic Balance Sheet

- The end-of-period total claim payments for the insurance company is

$$L = \sum_{k=1}^{K} L_k.$$

- The value of the liability, assuming full payment (no insurer credit risk), can be written as

$$V_L = E^Q \left[\frac{L}{1+r} \right] = \sum_{k=1}^{K} E^Q \left[\frac{L_k}{1+r} \right] = \sum_{k=1}^{K} V_{L_k}$$

so that Q measure prices all assets and liabilities at fair value (arbitrage-free).

Economic Balance Sheet

- Liability claim payments are still risky since the future pay-off is a random variable. Value of the liability in terms of real world or historical probabilities is

$$
\begin{aligned}
V_L &= E^P[mL] \\
&= E^P[m]\,E^P[L] + cov^P(m,L) \\
&= \frac{E^P[L]}{1+r} + cov^P(m,L)
\end{aligned}
$$

  where m is a stochastic discount factor. Requires no allocation of capital.

- This value of the liabilities allows for relevant economic risk factors but does not take into account the (credit risk) insolvency of the insurance company.

Fair rate of return

- Fair value of total liabilities is $V_L - D$ where D is the value of the insolvency exchange option for the insurer (a discount for credit risk) given by

$$D = \frac{E^Q\left[\max\left(L - A, 0\right)\right]}{1 + r} = \frac{E^Q\left[L - A \middle| \frac{A}{L} < 1\right] \Pr{}^Q\left[\frac{A}{L} < 1\right]}{1 + r}$$

- Insolvency exchange option value reflects both the probability of insolvency and the expected severity of the insolvency based on the risk neutral probabilities. For extreme events, assuming risk aversion, the risk neutral probabilities will usually exceed the actual historical or real world probabilities.

Solvency

- Denote the solvency ratio by s so that $V_A = (1 + s)\, V_L$. A solvent insurer will have $s > 0$.

- The market value of the initial actuarial surplus is given by

$$S = V_A - V_L > 0$$

where the asset values allow for the issuer default but the actuarial liability values of the insurer do not allow for the insurer insolvency (credit) risk.

Solvency

- The market value of the equity V_X will be the actuarial surplus plus the value of the insolvency exchange option. Since the market value of the equity is the market value of the assets less the market value of the liabilities we have

$$
\begin{aligned}
V_X &\equiv V_A - (V_L - D) \\
&= sV_L - D > 0
\end{aligned}
$$

- Allocating capital to lines of business implies allocation of V_A, V_L, and D.

- Allocation of V_L is clear, allocation of D is by payoffs by line and allocation of V_A is effectively an internal allocation.

Economic Balance Sheet

- End of the period payoffs on the balance sheet of the insurer

Balance Sheet	Initial Value	End of Period Payoff
Assets	V_A	A
Liabilities	$V_L - D$	$\left[\begin{array}{l} \min(L, A) \\ = L + \min(A - L, 0) \\ = L - \max(L - A, 0) \end{array}\right]$
Equity	$S + D$	$\left[\begin{array}{l} \max(A - L, 0) \\ = A - L - \min(A - L, 0) \\ = A - L + \max(L - A, 0) \end{array}\right]$

Establishing Fair Values

- At the start of the period the insurer sets its investment policy $\left(w_j \text{ for all } j\right)$, determines the liability risks that the company will underwrite and its solvency ratio, s. This information is assumed known and reflected in the valuation of cashflows.

- The distribution of liability risks, L, is known and the value of these liabilities ignoring the insurer default put option, V_L, is given by the risk neutral Q-probabilities, or equivalently the stochastic discount factor, since we assume a complete market.

- The total value of the assets is determined from the liability value and the solvency ratio $(1+s)\,V_L$.

Establishing Fair Values

- Given the distribution of both A and L, the value of the insolvency exchange option is

$$D = \frac{E^{Q}\left[\max\left(L - A, 0\right)\right]}{1 + r}$$

- The total fair (market) premiums for the policyholders is $V_L - D$, and the total capital (required from shareholders) is $V_A - (V_L - D)$.

Fair Rate of Return

- The premiums charged provide a competitive or "fair" rate of return to equity, allowing for the insolvency of the insurer, and the balance sheet structure is determined by the liabilities underwritten and the target solvency ratio.

- Capital earns a fair rate of expected return since all assets and liabilities, including the insolvency exchange option, are fairly priced under the risk neutral Q-measure. The fair rate of return reflects the leverage of the insurer balance sheet.

Capital Allocation to Lines of Business

- Capital consists of assets less liabilities adjusted for the insurer default put option $= V_A - (V_L - D)$. So to allocate capital we need to allocate assets, liabilities and insurer default put option to lines of business.

- Allocation of liabilities to line of business is straightforward. For line of business k the payoff is L_k and the allocated value is V_{L_k}.

- Total claims sum across the balance sheet as do fair values of liabilities so that $L = \sum_{k=1}^{K} L_k$ and $V_L = \sum_{k=1}^{K} V_{L_k}$.

Capital Allocation to Lines of Business

- For allocation of the insurer default put option, assume all lines of business rank equally in the event of default (other assumptions easily handled).

- Policyholders with claims due and payable in line of business k will be entitled to a share $\frac{L_k}{L}$ of the assets of the company where the total outstanding claim amount is $L = \sum_{k=1}^{K} L_k$.

- The end-of-period payoff to line of business k is well defined based on this equal priority as

$$\begin{array}{ll} \frac{L_k}{L} A & \text{if } L > A \text{ (or } \frac{L}{A} > 1) \\ L_k & \text{if } L \leq A \text{ (or } \frac{L}{A} \leq 1) \end{array}$$

Capital Allocation to Lines of Business

- Let the value of the insurer default put option (insolvency exchange option) allocated to line of business k be denoted by D_k, then this is given by the value of the pay-off to the line of business in the event of insurer default

$$D_k = \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right]$$

- Value of the insurer default put option for each line of business "adds up" to the total insurer default put value

$$\sum_{k=1}^{K} D_k = \frac{1}{1+r} \sum_{k=1}^{K} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] = \frac{1}{1+r} E^Q \left[\max \left[L - A, 0 \right] \right]$$

Capital Allocation to Lines of Business

- Allocation of assets to line of business is an internal insurer allocation with no economic impact on the by-line payoffs or risks of the insurer since assets are available to meet the losses of all lines of business.

- There are an infinite number of possible ways of allocating assets and hence of allocating capital to lines of business.

- Each allocation will correspond to a different expected return on allocated capital by line. There is a unique allocation that corresponds to an equal expected return on allocated capital for each line, equal to the company expected return on capital.

Capital Allocation to Lines of Business

- Allocation of assets to lines of business is "irrelevant" for fair pricing or solvency since we can do this without allocating capital.

- If we have to allocate capital to do fair pricing then we need to specify an expected return on capital by line that is consistent with the (internal) capital allocation used. We can not use Value at Risk to determine economic capital by line and then independently apply an expected return on equity to the line to determine fair prices. Allocated capital and expected return by line must be consistently determined.

- Allocation of the insurer default put option is required for fair (arbitrage-free) pricing by line of business so it is important to have analytical results.

Capital Allocation to Lines of Business

- Possible methods of allocating assets
  - Surplus allocated to lines of business so that each line of business has the same solvency ratio as for the total insurer
  - Assets allocated so that the expected return on allocated capital will be equal across all lines of business and the same as for the total insurer.
- Latter often used in practice and suggested by a number of researchers.

Market Premiums by Lines of Business

- Many authors - for example Phillips, Cummins and Allen (1998), Myers and Read (2001) - implicitly or explicitly assume that price by line of business is given by

$$V_{L_k} - D_k = V_{L_k} - \frac{V_{L_k}}{V_L} D$$

so that the insurer default put option value is allocated in proportion to the by line liability values

$$D_k = \frac{V_{L_k}}{V_L} D = \frac{E^Q[L_k]}{E^Q[L]} \frac{E^Q[\max(L - A, 0)]}{1 + r}$$

- Result holds if liabilities are deterministic.

Market Premiums by Lines of Business

- Myers and Read (2001) is a prize winning paper on capital allocation for insurers. Provides analytical expressions for log-normal and normal distribution assumptions that have been used by practitioners.

- In Myers and Read (2001) the insurer default put option value by line as a percentage of the by-line liability, denoted by d_k, is determined as a (marginal) **sensitivity (per unit of liability)**

$$d_k = \frac{\partial D}{\partial V_{L_k}}$$

Market Premiums by Lines of Business

- d_k is assumed to be the same for all lines of business in order for $\frac{D}{L}$ to be invariant for small changes in a line of business and this condition is used to determine surplus allocation by line in Myers and Read (2001). Thus

$$d_k = \frac{D}{V_L} \text{ for all } k$$

or

$$\frac{D_k}{V_{L_k}} = \frac{D}{V_L} \text{ or } D_k = \frac{V_{L_k}}{V_L} D \text{ for all } k$$

- Insurer default put option value is allocated in proportion to liability values by line. So this makes no sense?

Example - Discrete State, Discrete Time

- Single risky asset and two lines of business, assumed payoff for a unit of the risky and risk free asset and the payoff to the liabilities as well as the P and Q probabilities are

Table 1: Probabilities and Payoffs for Example Insurer

			Time 1 Payoffs			
State	P-probs	Q-probs	Risky Asset	Risk Free Asset	Liability 1	Liability 2
1	0.1	0.1	0.6	1.05	200	40
2	0.6	0.4	1.1	1.05	4	10
3	0.2	0.4	1.0	1.05	2	4
4	0.1	0.1	1.5	1.05	0	310
Time 0 Value			1.0	1.0	21.3333	38.6667

Example - Discrete State, Discrete Time

- Table 2 gives the payoffs for the assets and liabilities as well as the amount of liabilities not met because of insufficient assets. Note that the insurer defaults in both State 1 and State 4.

Table 2 Insurer Balance Sheet Payoffs					
	Time 1 Insurer Balance Sheet Payoffs				
State	Assets	L_1	L_2	Total L	$max(L - A, 0)$
1	120	200	40	240	120
2	220	4	10	14	0
3	200	2	4	6	0
4	300	0	310	310	10
Time 0 Value	200	21.3333	38.6667	60	12.381

Example - Discrete State, Discrete Time

- The surplus ratio for the insurer is

$$s = \frac{S}{L} = \frac{200 - 60}{60} = 2.3333$$

- The economic capital of the insurer at time 0 will be the value of the assets less the value of the liabilities ignoring the insolvency costs and plus the value of the insolvency exchange option which is $200 - 60 + 12.381 = 152.381$.

Example - Discrete State, Discrete Time

- Shortfall in the event of insolvency for each line of business are given in Table 3.

Table 3 Liability Shortfalls in the Event of Insolvency		
	Time 1 Liability Shortfalls	
State	$D_1 = L_1 \max(1-\frac{A}{L}, 0)$	$D_2 = L_2 \max(1-\frac{A}{L}, 0)$
1	100	20
2	0	0
3	0	0
4	0	10
Time 0 Value	9.5238	2.8571

Example - Discrete State, Discrete Time

- The allocation of the insurer shortfall of assets over liabilities is based on equal priority of the policyholders to the assets for each line of business. Thus in State 1 the shortfall of 120 is allocated in proportion to the outstanding liabilities so that $\frac{200}{240} \times 120 = 100$ is the shortfall for line of business 1 and $\frac{40}{240} \times 120 = 20$ is the shortfall for line of business 2.

- The premium for each line of business is determined allowing for the insurer insolvency exchange option value. For line of business 1 the premium will be $21.3333 - 9.5238 = 11.8095$ and for line of business 2 it will be $38.6667 - 2.8571 = 35.8095$.

Example - Discrete State, Discrete Time

Table 4 gives the insurer equity payoffs

Table 4 Insurer Equity Payoffs				
	Time 1 Insurer Equity Payoffs			
State	P-probs	Assets	Total L	Equity $= max(A - L, 0)$
1	0.1	120	240	0
2	0.6	220	14	206
3	0.2	200	6	194
4	0.1	300	310	0
Time 0 Value		200	60	152.3810

Example - Discrete State, Discrete Time

- The ratio of the insolvency exchange option value to the value of the liabilities for the insurer and for each line of business is

$$d = \frac{D}{V_L} = \frac{12.381}{60} = 0.2063$$

$$\begin{aligned} d_1 &= \frac{D_1}{V_{L_1}} = \frac{9.5238}{21.3333} = 0.4464 \\ d_2 &= \frac{D_2}{V_{L_2}} = \frac{2.8571}{38.6667} = 0.0739 \end{aligned}$$

The expected return to equity for the insurer is

$$\frac{0.1 \times 0 + 0.6 \times 206 + 0.2 \times 194 + 0.1 \times 0}{152.3810} - 1 = 0.06575$$

Example - Discrete State, Discrete Time

- Allocate the capital to lines of business to equate the expected return to capital by line of business and to the insurer expected return to equity of 0.06575.

- Requires an allocation of 50.3544 of the assets to line of business 1 and 149.6456 to line of business 2 to equate the expected return to capital (equity) for each line of business.

- This gives a capital allocation (assets - liabilities + put) of $50.3544 - 21.3333 + 9.5238 = 38.5449$ to line of business 1 and $149.6456 - 38.6667 + 2.8571 = 113.8361$ to line of business 2. But line of business 1 is more risky?

Continuous Time Single Period Model - Closed Form Derivation (similar assumptions to Myers-Read)

- We will use notation in Sherris and van der Hoek (2006) - some differences from earlier slides, $V = V(0)$, $L = L(0)$, $D = D(0)$ and $S = V - L$.

- The insurer economic (or fair value) balance sheet is as follows:

Balance Sheet	**Initial Value**	**End of Period Payoff**
Assets	V	$V(T)$
Liabilities	$L - D$	$L(T) - (L(T) - V(T))^+$
Equity	$S + D$	$\left[V(T) - L(T) + (L(T) - V(T))^+\right]$

Table 1: *Total Insurer Economic Balance Sheet*

Continuous Time Single Period Model - Closed Form Derivation (similar to Myers-Read)

- Risk measure is the insolvency exchange option value

$$
\begin{aligned}
D(0) &= E^Q\left[e^{-rT}\left[L(T)-V(T)\right]^+ \mid \mathcal{F}_0\right] \\
&= E^Q\left[e^{-rT}\left[L(T)-V(T)\right] \mid \frac{V(T)}{L(T)}<1\right]\Pr{}^Q\left[\frac{V(T)}{L(T)}<1\right]
\end{aligned}
$$

Internal By Line of Business Balance Sheet

Balance Sheet	Initial Value	End of Period Payoff
Assets	V_i	$V_i\,(T)$
Liabilities	$L_i - D_i$	$L_i\,(T) - L_i\,(T)\left(1 - \frac{V(T)}{L(T)}\right)^+$
Equity	$S_i + D_i$	$\left[V_i\,(T) - L_i\,(T) + L_i\,(T)\left(1 - \frac{V(T)}{L(T)}\right)^+\right]$

Table 2: *Internal Balance Sheet for Line of Business i*

Comments on Internal By Line of Business Balance Sheet

- Assumes equal priority to assets in the event of insolvency for outstanding liabilities by line of business

- Allocation of surplus is arbitrary (capital allocation irrelevance) - only an internal allocation and many different allocations will "add up"

- Allocation of insurer default put option value (insolvency exchange option value) reflects the by-line loss in the event of insolvency and has economic significance (pricing, fair valuation).

Capital Allocation - Insurer Default Option Value

- Assumptions

$$dL_i(t) = \mu_i L_i(t)\,dt + \sigma_i L_i(t)\,dB^i(t) \quad \text{for } i = 1, \ldots, M$$

$$\Lambda(t) \triangleq \frac{V(t)}{L(t)}$$

$$d\Lambda(t) = \mu_\Lambda \Lambda(t)\,dt + \sigma_\Lambda \Lambda(t)\,dB^\Lambda(t)$$

- Allows derivation of closed form expression for default option value by line of business.

Capital Allocation - Insurer Default Option Value

- End-of-period payoff to line of business i is well defined with equal priority

$$\frac{L_i(T)}{L(T)} V(T) \quad \text{if } L(T) > V(T) \text{ (or } \frac{V(T)}{L(T)} \leq 1\text{)}$$
$$L_i(T) \quad \text{if } L(T) \leq V(T) \text{ (or } \frac{V(T)}{L(T)} > 1\text{)}$$

$$\begin{aligned} D_i(t) &= E^Q \left[e^{-r(T-t)} L_i(T) \left[1 - \frac{V(T)}{L(T)} \right]^+ |\mathcal{F}_t \right] \\ &= E^Q \left[e^{-r(T-t)} L_i(T) \left[1 - \Lambda(T) \right]^+ |\mathcal{F}_t \right] \end{aligned}$$

Capital Allocation - Insurer Default Option Value "Adds Up"

$$
\begin{aligned}
\sum_{i=1}^{M} D_i(t) &= \sum_{i=1}^{M} E^Q\left[e^{-r(T-t)} L_i(T)\left[1-\Lambda(T)\right]^+ |\mathcal{F}_t\right] \\
&= \sum_{i=1}^{M} E^Q\left[e^{-r(T-t)} L_i(T)\left[1-\frac{V(T)}{L(T)}\right]^+ |\mathcal{F}_t\right] \\
&= E^Q\left[e^{-r(T-t)}\left[L(T)-V(T)\right]^+ |\mathcal{F}_t\right] \\
&= D(t)
\end{aligned}
$$

Insurer Default Option Value By Line - Closed Form Expression

- Change of measure (Radon-Nikodym derivative)

$$\frac{dQ^i}{dQ}|\mathcal{F}_T = Z_i(t) = \frac{1}{L_i(0)}\frac{L_i(t)\, e^{(r-\mu_i)t}}{e^{rt}}$$

- Value becomes

$$D_i(0) = L_i(0)\, e^{\mu_i T} E^{Q_i}\left[e^{-rT}\left(1 - \Lambda(T)\right)^+ |\mathcal{F}_0\right]$$

Insurer Default Option Value By Line - Closed Form Expression

- Exchange option closed form (used to compute by-line allocation using change of measure)

$$\begin{aligned} M(0) &= E^Q\left[e^{-rT}\left[1-\Lambda(T)\right]^+ |\mathcal{F}_0\right] \\ &= e^{-rT}N(-d_{20}) - \Lambda(0)\, e^{-(r-\mu_\Lambda)T}N(-d_{10}) \end{aligned}$$

$$d_{10} = \frac{\ln\Lambda(0) + \left(\mu_\Lambda + \frac{1}{2}\sigma_\Lambda^2\right)T}{\sigma_\Lambda\sqrt{T}}$$

$$d_{20} = d_{10} - \sigma_\Lambda\sqrt{T}$$

Insurer Default Option Value By Line - Closed Form Expression

$$\begin{aligned} D_i(0) &= L_i(0)\, e^{\mu_i T} E^{Q_i}\left[e^{-rT}(1-\Lambda(T))^+ \,|\mathcal{F}_0\right] \\ &= L_i(0)\, e^{\mu_i T} M^i(0) \\ D(0) &= \sum_{i=1}^{M} D_i(0) \end{aligned}$$

$M^i(0)$ evaluate with $M(0)$ formula - μ_Λ replaced by $\mu_\Lambda^i = \mu_\Lambda + \rho_{i\Lambda}\sigma_i\sigma_\Lambda$
$d\Lambda(t) = \mu_\Lambda \Lambda(t)\, dt + \sigma_\Lambda \Lambda(t)\left(d\widetilde{B}^\Lambda(t) + \rho_{i\Lambda}\sigma_i dt\right)$
$\widetilde{B}^\Lambda(t)$ Brownian motion under Q_i and $dB^i(t)\, dB^\Lambda(t) = \rho_{i\Lambda} dt$

Comments on Capital Allocation - Myers and Read (2001) with constant d_i

- Myers and Read (2001) results are sensitivities to maintain the default option value per unit of liabilities and should not be used as the default put option value ($d_i = \frac{\partial D}{\partial L_i}$)

- Note $\frac{\partial}{\partial L_i}\left(\frac{D}{L}\right) = \frac{1}{L}\left[\frac{\partial D}{\partial L_i} - \frac{D}{L}\right]$ and selecting s_i, the by-line surplus as a percentage of L_i, so that $\frac{\partial D}{\partial L_i} = \frac{D}{L}$ for all i ensures $\frac{\partial}{\partial L_i}\left(\frac{D}{L}\right) = 0$.

- Constant d_i required to maintain balance sheet $\frac{D}{L}$ and s_i gives **incremental** capital required from shareholders for small changes in each line.

Sensitivities and Capital Allocation

- If w_i is the weight in line of business i then for any risk measure that is an homogeneous function of order one in the amounts in the risks so that $RM\left(kw_1, kw_2, \ldots, kw_n\right) = kRM\left(w_1, w_2, \ldots, w_n\right)$ we have

$$RM = \sum_{i=1}^{n} w_i \frac{\partial RM}{\partial w_i}$$

from Euler's homogeneous function theorem.

- So capital "adds up" for such a risk measure but this is a purely mathematical result.

Example from Myers and Read (2001)

		Ratio to Liabilities	Standard Deviation	Correlations		
				Line 1	Line 2	Line 3
Line 1	$100	33%	10%	1.00	0.50	0.50
Line 2	$100	33%	15%	0.50	1.00	0.50
Line 3	$100	33%	20%	0.50	0.50	1.00
Liabilities	$300	100%	12.36%	0.74	0.81	0.88
Assets	$450	150%	15%	-0.20	-0.20	-0.20
Surplus	$150	50%				

Table 3: *Data from Table 2 of Myers and Read*

Example from Myers and Read (2001)

		Covariance with Liabilities	Covariance with Assets	μ_{Λ}^{i}
Line 1	$100	0.0092	-0.0030	0.0076
Line 2	$100	0.0150	-0.0045	0.0003
Line 3	$100	0.0217	-0.0060	-0.0079
Liabilities	$300	0.0153	-0.0045	0.0000
Assets	$450		0.0225	
Surplus	$150		σ_{Λ}	0.2163

Table 4: *Parameters for Table 2 Data of Myers and Read*

Example from Myers and Read (2001)

	Partial Derivatives Uniform Default Value Myers-Read/Sherris-van der Hoek Comparison			
	MR d_i %	MR s_i%	SvdH d_i%	SvdH s_i%
Line 1	0.3112	37.75	0.3119	37.53
Line 2	0.3112	49.55	0.3119	49.50
Line 3	0.3112	62.90	0.3119	62.98
Total	0.3112	50	0.3119	50.00

Table 5: *Line by Line Sensitivities Table 2 Data of Myers and Read - Lognormal assumption*

Note: s_i are sensitivities (not allocations) to maintain value of d with $d_i = d$

Example from Myers and Read (2001)

	Capital Allocations Uniform Default Value Myers-Read/Sherris-van der Hoek Comparison			
	MR d_i %	MR s_i%	SvdH $\widetilde{d}_i$%	SvdH $\widetilde{s}_i$%
Line 1	0.3112	37.75	0.2852	not unique
Line 2	0.3112	49.55	0.3102	not unique
Line 3	0.3112	62.90	0.3404	not unique
Total	0.3112	50	0.3119	50

Table 6: *Line by Line Allocations Table 2 Data of Myers and Read - Lognormal assumption*

Data from Panjer (2001)

Line	Amount	PerCent	Standard Deviation (Dollar)	Standard Deviation (Percent)
Line 1	36.00	9.6	2.69	7.47
Line 2	120.40	32.3	4.49	3.73
Line 3	1.30	0.3	0.21	16.12
Line 4	52.42	14.0	1.32	2.51
Line 5	0.70	0.2	0.57	82.14
Line 6	48.09	12.9	3.87	8.05
Line 7	47.40	12.7	1.59	3.36
Line 8	8.08	2.2	0.96	11.85
Line 9	8.64	2.3	1.06	12.29
Line 10	50.15	13.4	2.59	5.17
Liabilities	373.18	100	6.73	1.80
Assets	400.42	107	60.06	15.00
Surplus	27.24	7		

Table 7: *Liabilities and Standard Deviations*

Data from Panjer (2001)

Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9	Line 10
1.00	0.00	0.12	-0.02	0.18	-0.26	-0.12	0.11	0.08	-0.03
0.00	1.00	0.05	0.27	0.02	0.08	0.16	-0.21	-0.17	-0.15
0.12	0.05	1.00	0.01	-0.11	0.10	0.03	-0.12	-0.09	-0.12
-0.02	0.27	0.01	1.00	0.22	0.05	0.09	-0.11	0.13	-0.23
0.18	0.02	-0.11	0.22	1.00	-0.11	0.01	-0.03	0.14	-0.01
-0.26	0.08	0.10	0.05	-0.11	1.00	0.07	-0.09	-0.46	-0.16
-0.12	0.16	0.03	0.09	0.01	0.07	1.00	-0.25	0.08	0.14
0.11	-0.21	-0.12	-0.11	-0.03	-0.09	-0.25	1.00	-0.16	-0.16
0.08	-0.17	-0.09	0.13	0.14	-0.46	0.08	-0.16	1.00	0.21
-0.03	-0.15	-0.12	-0.23	-0.01	-0.16	0.14	-0.16	0.21	1.00
0.25	0.69	0.09	0.35	0.16	0.40	0.39	-0.18	-0.08	0.18

Table 8: *Correlations by line of business for Panjer Data*

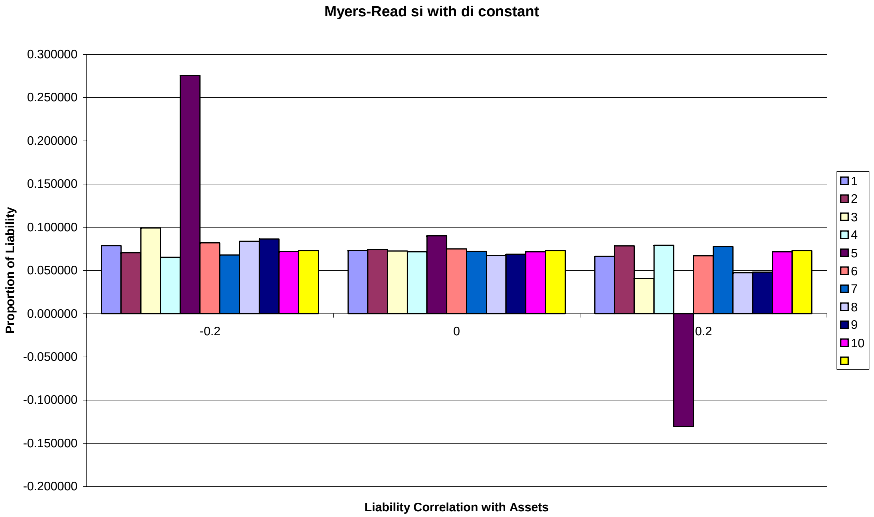

Figure 1: Myers and Read s_i for Varying Correlations between Liability Lines and Assets

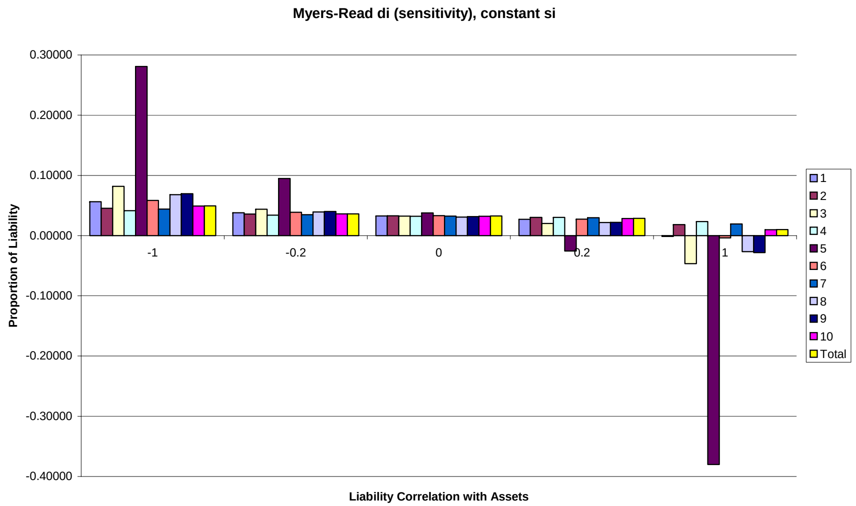

Figure 2: Myers and Read d_i values for constant s_i with varying Correlations between Liability Lines and Assets

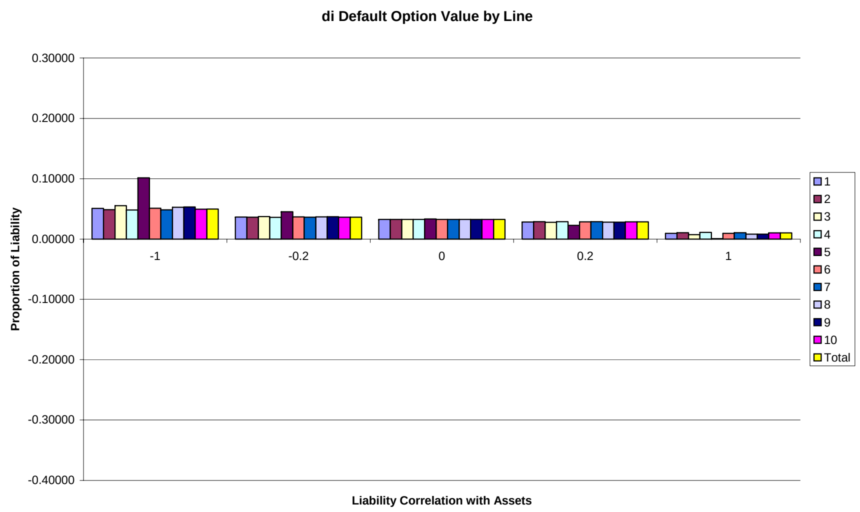

Figure 3: Default Option Values for Varying Correlations Between Liability and Assets

Conclusions

Capital (surplus) allocation is "irrelevant" in the complete markets arbitrage-free model with no frictions since it is not required for financial decision making (pricing, risk management).

Allocation of the insolvency exchange option is not "irrelevant" and is required for fair rate of return by-line pricing and financial decision making.

Closed form expressions are given for the by-line and total insolvency exchange option value assuming arbitrage-free and complete markets.

Myers and Read (2001) results are sensitivities (capital required to maintain ratio of insurer default option value to liabilities for small changes to a single line of business) and not insurer default option capital allocations.