Rolling Hills

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July 2007

The problem (i)

- Can’t tell you yet
- Stylised problem discussed here
- Standard approach used for consulting but this amusing viewpoint was tangential

The problem (stylised)

- How much capital for extreme stock price movements?
- Can we get a good model of extreme returns?
  - Extreme returns rare - need lots of data
  - Distributions change over time
  - Fat tails

Stage one

- Reduce fat by conditional variance
- Model variance by GARCH
  - log return at time t := r_t ,
  - standardised return := s_t,
  - volatility estimate at time t-1 := σ_{t-1} .
- $s_t = r_t / \sigma_{t-1}$
- $\sigma^2_t = k\sigma^2_m + j\sigma^2_{t-1} + (1-j-k)s_t^2$
- σ^2_m is a long term central value to which variance of return reverts at a speed related to k.
- We are assuming the average return is zero over small time intervals to avoid estimating the nuisance parameter of the mean return

Stage one (ii)

- Standardised returns are still fat-tailed
- Skew is close to zero
- Overall std dev is close to 1.0
- Model the tails

EVT – a quick intro

- Tails of distribution of returns are Generalised Pareto Distribution (GPD)
- $F(x) = 1 - (1 + kx/s)-1/k$
- *x* is sufficiently large and positive, it is presumed to be larger than some threshold *u*

EVT

- $s > 0$ and is a scale parameter – the larger the value of *s,* then the more spread out are the values of *x*
- *k* is a shape parameter – as *k* decreases (*1/k* increases), the tail of the distribution becomes fatter
- $1+kx/s > 0$
- **No standard notation available**

Shape parameter

- Hill estimate is used for the shape parameter:
- $k = \Sigma ln(x/u) / n$
- where:
- *n* is the number of observations where *x* is more extreme than *u,*
  - summation is only over those extreme values

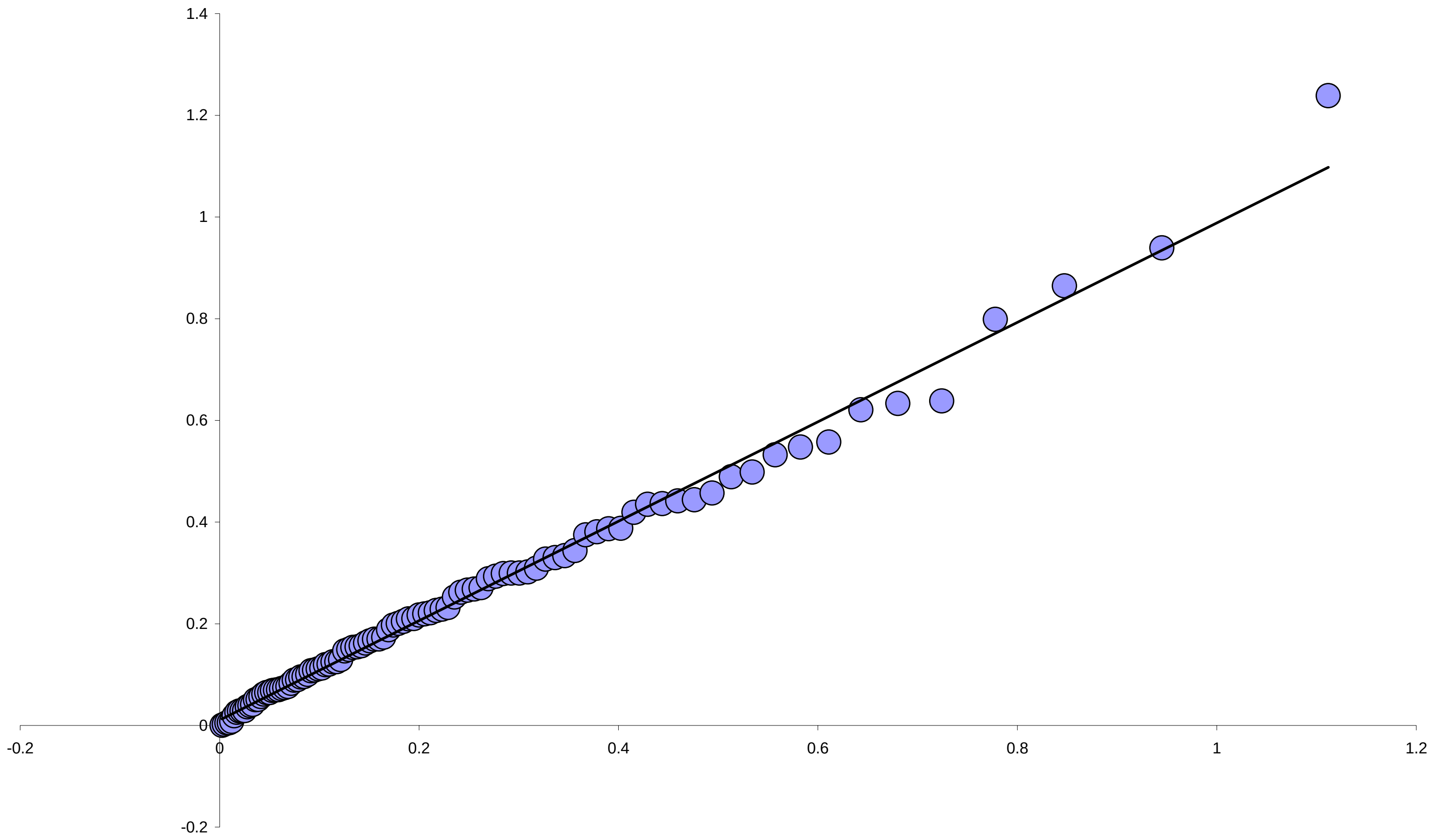
Scale parameter

- $s = k(\Sigma(x-u)^k / n)^{1/k}$
- The shape parameter governs how fast the tail decays – like $x^{-1/k}$
- The scale parameter gives us a measure of the typical size of the exceedances
- Both are important in the determination of the VaR

Quantile Plot (Q Plot)

- If GPD is good, log relative rank versus log exceedance should be a straight line.
- Q Plot drawn to enable judgement
- We may only have a few observations in the tail, and so not much statistical power – eyeballing may be the best test.

BHP QQ plot



Value at Risk

- Let p be the probability that an observation is in the tail.
  - probability of value more extreme than u is p
- the percentile at probability level q is
- $VaR_q = u + T_q$
- $T_q = (s/k)\ [(q\ /p)^{-k}-1]$
- For good estimates need $q << p$.

Typical uses

- Pick a tail probability like 5% or 2.5%
- Estimate 1% Var
- Estimate expected shortfalls

Problems

- How do parameters change?
- Every new data point:
  – Re-estimate percentiles
- Give recent data extra weight?
  – Difficult to calculate percentiles

Exponential weighting

- Typically used as a simple GARCH process
- $\sigma^2_t = k\sigma^2_m + j\sigma^2_{t-1} + (1-j-k)s_t^2$

EW-EVT

- Model tail standardised log returns
- Don’t choose tail by *p* eg $u=F^{-1}(2.5\%)$
  – Choose tail by cutoff point eg u=-2
- Tail probability is now variable
- Choose long decay factor
  – 0.996 → avg age = 250 days

Tail probability

- $p_t = jp_{t-1} + (1-j)I_t$
- I_t is:
  - 1 if we're in the tail
  - 0 otherwise
- probability goes up if we're in tail, goes down otherwise

Shape parameter

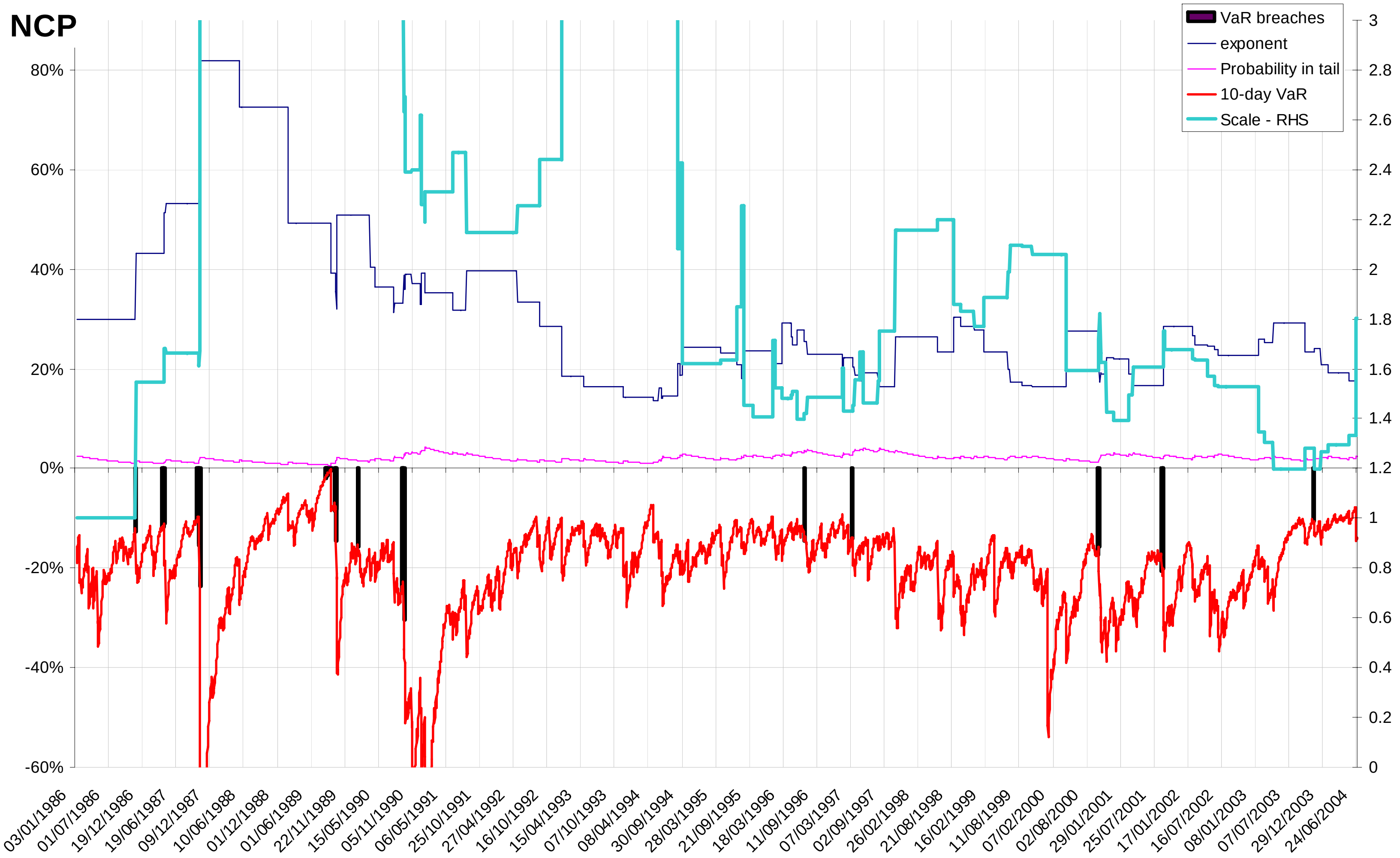
- $k = \Sigma ln(x/u) / n$ usual formula
- $K_t = jK_{t-1} + (1-j)I_t \, ln(x/u)$
- $k_t = K_t / p_t$
- shape parameter only changes when we get a tail value

Scale parameter

- $S_t = \sum^n{}_1 \; ln(x/u)^{k(i)} \; (1\text{-}j)j^{(n\text{-}i)} I_i \; + \; s_0 j^n$
- $s_t = S_t \,/\, p_t$
- Scale parameter only changes when we're in the tail

EW-VaR

- $VaR_q = u + T_q$
- $T_q = (s/k)\ [(q\ /p)^{-k}-1]$
- new value calculate at each point
- this can be put in spreadsheet


NCP
VaR breaches
exponent
Probability in tail
10-day VaR
Scale - RHS
80%
60%
40%
20%
0%
-20%
-40%
-60%
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
03/01/1986
01/07/1986
19/12/1986
19/06/1987
09/12/1987
10/06/1988
01/12/1988
01/06/1989
22/11/1989
15/05/1990
05/11/1990
06/05/1991
25/10/1991
27/04/1992
16/10/1992
15/04/1993
07/10/1993
08/04/1994
30/09/1994
28/03/1995
21/09/1995
18/03/1996
11/09/1996
07/03/1997
02/09/1997
26/02/1998
21/08/1998
16/02/1999
11/08/1999
07/02/2000
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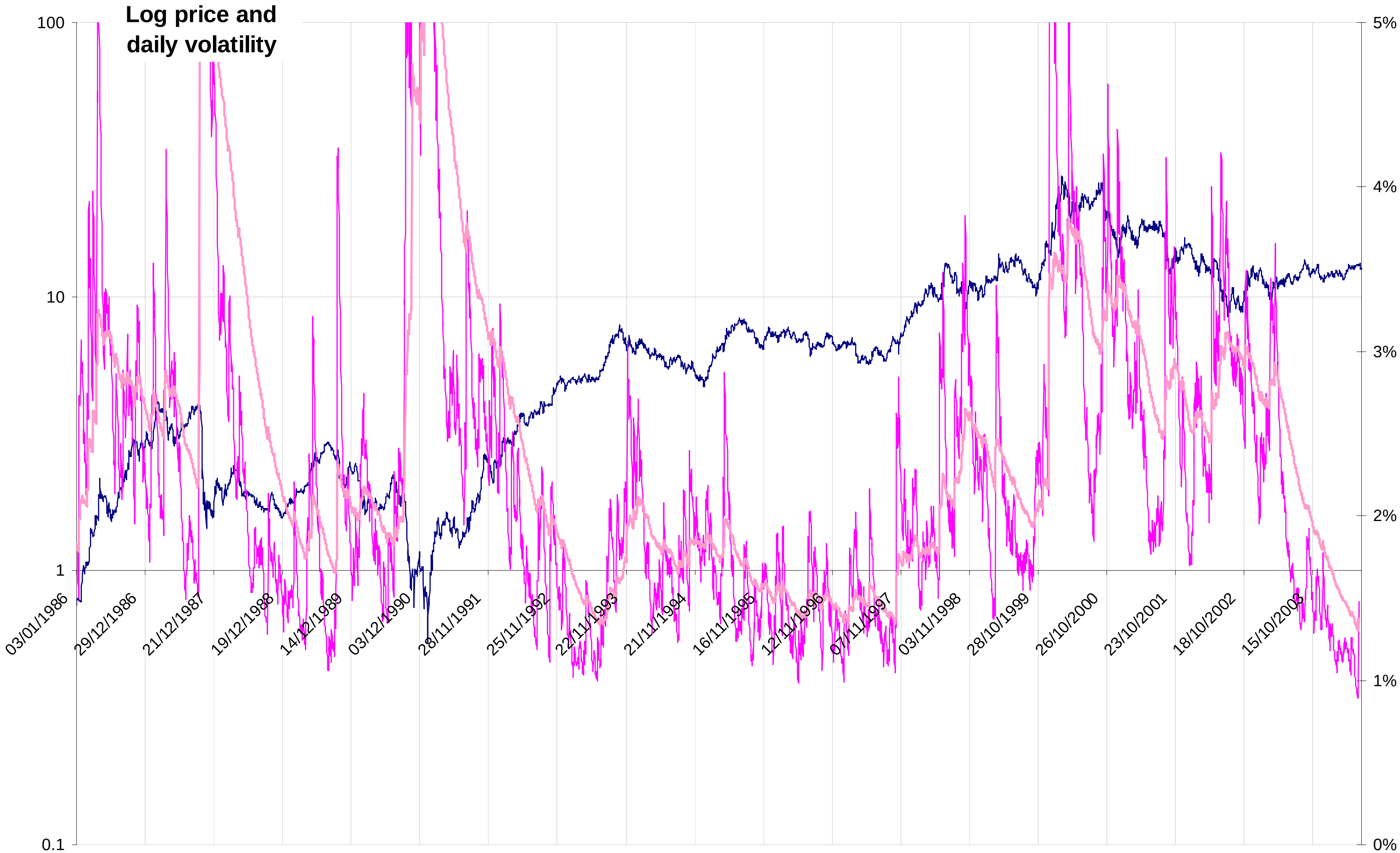
29/12/2003
24/06/2004


Log price and daily volatility
100
10
1
0.1
5%
4%
3%
2%
1%
0%
03/01/1986
29/12/1986
21/12/1987
19/12/1988
14/12/1989
03/12/1990
28/11/1991
25/11/1992
22/11/1993
21/11/1994
16/11/1995
12/11/1996
07/11/1997
03/11/1998
28/10/1999
26/10/2000
23/10/2001
18/10/2002
15/10/2003

Observations

- VaR is inadequate when $p \approx q$
  - see late 1989
- Shape parameter is much more stable than scale
- VaR is variable

Uses

- Gives a feel for how EVT estimates are variable
- Possible explanatory power for company risk

Bibliography

- McNeil, Alexander, 1999, Extreme Value Theory for Risk Managers., for an introduction.
- Alexander, C. and E. Sheedy, 2007. *Model-Based Stress Tests: Linking Stress Tests to VaR for Market Risk* (ICMA Centre Discussion Paper in FinanceDP2007-02)