Overview of Credit Derivatives and Credit Risk Modelling

Erik Schlögl

Erik.Schlogl@uts.edu.au

Credit Risk

“Default risk is the risk that an obligor does not honour his payment obligations.”

Typically,

- Default events are rare.
- They may occur unexpectedly.
- Default events involve significant losses.
- The size of these losses is unknown before default.

All payment obligations represent some sort of default risk.

Components of Credit Risk

- **Arrival risk** — whether default will occur or not
  $\Rightarrow$ probability of default

- **Timing risk** — when default occurs

- **Recovery risk** — how severe the losses are
  $\Rightarrow$ probability distribution of the recovery rate

- **Market risk** — changes in the market price of a defaultable asset
  $\Rightarrow$ risk change risk, market price correlation risk

- **Default correlation risk** — risk of several obligors defaulting
  $\Rightarrow$ joint arrival risk, joint timing risk

Credit Derivatives

First cut definition:

- A derivative security that is primarily used to transfer, hedge or manage credit risk.

- A derivative security whose payoff is substantially affected by credit risk.

Credit Derivatives

Narrower definition:

- A **credit derivative** is a derivative security that has a payoff which is conditioned on the occurrence of a **credit event**.

- The credit event is defined with respect to a **reference credit**, and the **reference credit asset(s)** issued by the reference credit.

- If the credit event has occurred, the **default payment** has to be made by one of the counterparties.

- Besides the default payment, a credit derivative can have further payoffs that are not default contingent.

Market Terminology

- Buying a credit derivative typically means **buying credit protection**, which is economically equivalent to **shorting the credit risk**.
- So **selling** credit protection means going **long** the credit risk.
- Alternatively, one may speak of protection buyers/sellers as the payers/receivers of the premium.

Credit Events

- bankruptcy
- failure to pay
- obligation default
- obligation acceleration
- repudiation/moratorium
- restructuring
- ratings downgrade below given threshold
- changes in the credit spread

Underlying Assets

- loans
  - floating or fixed rate
  - may include optionality (interest rate caps, credit facilities)
  - not traded, thus recovery rate may be hard to determine
- bonds
  - fixed–coupon or floater
  - zero coupon
  - convertible
- counterparty risk

Uses for Credit Derivatives

- Traditional uses of derivatives: hedging, speculation, arbitrage
- Reduction of regulatory capital — this in particular applies to synthetic securitisations
- An unfunded way to diversify revenue

Asset Swaps

The aim is to swap the coupon of a defaultable bond into a cashflow of LIBOR plus a spread.

A typical asset swap package consists of the following payoffs:

- **A** sells to **B** for 1 (the notional value of the **C**–bond):
  - a fixed coupon bond issued by **C** with coupon $\overline{c}$ payable at coupon dates T_i, $i = 1, \ldots, N$
  - a fixed for floating swap

- The payments of the swap at each coupon date T_i are
  - **B** pays to **A**: $\overline{c}$, the amount of the fixed coupon of the bond
  - **A** pays to **B**: LIBOR + s^A

s^A is called the **asset swap spread**.

Asset Swaps (cont'd)

- An asset swap is essentially a **synthetic floating rate note**.
- Consisting of the purchase of an asset and a position in an interest rate swap.
- The asset swap buyer takes on an exposure (contingent on default) to the mark–to–market of an interest rate swap.
- Due to the small sensitivity to interest rate changes, the asset swap transforms a fixed rate bond into a **pure credit play**.
- Counterparty risk needs to be considered.

Total Return Swaps

The aim is to swap the **actual return** of a defaultable bond into a cashflow of LIBOR plus a spread.

Typical payoffs of a TRS:

A pays to **B** at regular intervals

- The coupon $\bar{c}$ (if there was one) of the bond issued by **C**
- The price appreciation $[\overline{C}(T_{i+1})-\overline{C}(T_i)]^+$ of the bond **C**
- The principal repayment of bond **C** (at the final payment date)
- The recovery value of the bond (if there was a default)

B pays

- A regular fee of LIBOR $+ s^{\text{TRS}}$
- The price depreciation $[\overline{C}(T_i) - \overline{C}(T_{i+1})]^+$
- The par value of the bond (if there was a default)

Credit Default Swaps

The aim is to transfer **only** the default risk from **A** to **B**.

The protection seller **B** agrees to pay the default payment

$$\text{notional} \times (1 - \text{recovery rate})$$

to **A** if a default has happened.

For this, **A** pays a periodic fee $\bar{s}$ to **B** (until maturity of the CDS or until default, whichever comes first)

Information Defining a CDS

- the reference obligor and his reference assets
- the definition of the credit event that is to be insured (default definition)
- the notional of the CDS
- the start of the CDS, the start of the protection
- the maturity date
- the credit default swap spread
- frequency and day count convention for the spread payments
- the payment at the credit event and its settlement

CDS Terminology

The asset that is traded in a CDS is **default protection**.

- A **long position** in a CDS is a position as protection buyer.

- A **short position** in a CDS is a position as protection seller.

- A **bid** of $x$bp on a CDS means that the bidder is willing to enter a CDS as protection buyer at a spread of $x$bp; conversely an **offer** of $x$bp means he is willing to act as protection seller.

Default Digital Swap

Fixed cash payment in default, usually the notional amount of the DDS.

If both CDS and DDS prices are available, we can imply a recovery rate. Compare the following portfolio positions:

1. A long position in a CDS with notional of 1, fee $\overline{s}$. Payoff in default: $1-$ Recovery.

2. A long position in a DDS with notional of $\overline{s}/\overline{s}^{\text{DDS}}$, fee $\overline{s}/\overline{s}^{\text{DDS}} \times \overline{s}^{\text{DDS}} = \overline{s}$. Payoff in default: $\overline{s}/\overline{s}^{\text{DDS}}$.

Since the cashflows before default are identical, we also must have

$$
\begin{aligned}
1 - \text{Recovery} &= \frac{\overline{s}}{\overline{s}^{\text{DDS}}} \\
\Leftrightarrow \qquad \text{Recovery} &= \frac{\overline{s}^{\text{DDS}} - \overline{s}}{\overline{s}^{\text{DDS}}}
\end{aligned}
$$

Other Credit Derivatives

- Rating–triggered credit default swaps
- Options on defaultable bonds
- Credit spread options
  - assert swap spread based options: asset swaptions
  - CDS spread based options: default swaption
  - yield–spread based options on defaultable bonds

One application is as a hedge for committed lines of credit
$\longrightarrow$ synthetic lending facilities.

Default Correlation Products

- Individual defaults in a large portfolio are expected, diversified and therefore typically do not require hedging.
- The real issue is a clustering of defaults or joint defaults $\rightarrow$ here default correlation becomes significant.
- However, first–to–default protection on a portfolio can appeal to some investors, for example to raise the credit quality of a portfolio to investment grade.

First–to–Default Swaps

- basket of reference credits $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_I$
- fee $\overline{s}^{\text{FtD}}$
- default event is the first default of any of the reference credits
- the FtD swap is terminated after the first default event
- the default payment is $1 -$ Recovery on the defaulted obligor

Collaterised Bond Obligations

- underlying portfolio of defaultable bonds
- the portfolio is transferred to an SPV
- the SPV issues notes
  - an equity (or first loss) tranche
  - several mezzanine tranches
  - a senior tranche
- if during the life of the CBO one of the bonds defaults, the recovery payments are reinvested in default–free securities
- at maturity of the CBO, the portfolio is liquidated and the proceeds distributed to the tranches, according to their seniority ranking

Refinements

- Other underlying portfolios: CLOs, CMOs
- Synthetic CDOs made up of credit default swaps
- Active vs. passive management: market value CDOs vs. cashflow CDOs
- “Black Box” CDOs

Loss Layers

- **Reference credit portfolio** consisting of I credit default swaps on the reference credits $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_I$ with notional amounts $K_1, K_2, \ldots, K_I$. The total notional amount is

$$K_{\text{tot}} = \sum_{i=1}^{I} K_i$$

- **Loss layer**, defined by lower and upper notional bounds K_L and K_U. The initial notional amount of the loss layer is $K = K_U - K_L$.

- **Credit events** are default events of the reference credits.

- **Cumulative loss** calculated as
  - the initial cumulative loss is $C_0 = 0$
  - at the j–th credit event, the cumulative loss is increased as

$$C_j = C_{j-1} + K_j(1 - \text{Recovery})$$

- the **cumulative loss of the loss layer** C^L is the amount by which the cumulative loss has hit the loss layer:

$$C_j^L = \min(\max(C_j - K_L, 0), K_U - K_L)$$

- The **default payment** of **B** to **A** at a default event is the increase in the cumulative loss of the loss layer.

- The notional of the loss layer is reduced by the default payments. If the notional reaches zero, the contract is unwound.

- **A** pays a periodic protection fee on the notional to **B**.

Structural (Asset–based) Models

Black and Scholes (1973) and Merton (1974):

Shares and bonds are derivatives on the firm's assets.

Limited liability gives shareholders the option to abandon the firm, to *put it to the bondholders*.

Bondholders have a short position in this put option.

Conversely, one can see equity as a call option on the value of the firm, with strike equal to the notional of the outstanding debt.

Assuming geometric Brownian motion for the firm's value results in a Black/Scholes–type formula for equity.

Key Assumptions and Limitations

- Can observe (or imply) the firm's value V

- The firm's value follows a lognormal random walk
  Can be relaxed at the cost of tractability (e.g. Zhou).

- Only zero-coupon debt
  Can be relaxed at the cost of tractability. (e.g. Geske)

- Default only at T: *Easy to relax. See Black-Cox and others.*

- Constant interest-rates r
  Very simple to generalise if there is independence between interest rates and V. Otherwise see Briys/de Varenne and Longstaff/Schwartz.

Extensions:

- Default before maturity T of the bonds.
- Default-free interest rates r stochastic.
- Different Securities: Coupon-Bonds, Callable Bonds, Convertibles . . . .
- More details in capital structure: Multiple Claims Maturity, Seniority . . . .

Model can be extended in analogy to equity option models:

BSM model for default risk	$\leftrightarrow$	BSM model for equity options
Early default	$\leftrightarrow$	Barrier options
complicated capital structure	$\leftrightarrow$	exotic options with complicated payoff functions
callability, convertibiliy	$\leftrightarrow$	American or Bermudan options

Can use the same numerical methods in both approaches.

Advantages of Firm's Value Models

- ✔ Relationships between different securities of same issuer
- ✔ Convertible bonds
- ✔ Collateralized Loans
- ✔ default correlation between different issuers can be modelled realistically.
- ✔ Fundamental orientation
- ✔ well-suited for theoretical questions (corporate finance)

Disadvantages of Firm’s Value Models

- ✘ observability of firm’s value: calibration, fitting
- ✘ bonds are not inputs but outputs
  defaultable bonds are far from being fundamentals
- ✘ all data is rarely available
- ✘ sovereign issuers cannot be priced
- ✘ often complex and inflexible
- ✘ unrealistic short-term spreads

Credit Ratings: Transition Matrix Models

The Transition Matrix:

P^1_{ij} is the probability of a ratings change from class i to class j within *one* year.

- Firm ABC has rating A
- There are 3 ratings: A, B and D (=Default)
- The rating agency publishes the following transition matrix:

	A		B		D	
A	p_{AA}	$= 0.80$	p_{AB}	$= 0.15$	p_{AD}	$= 0.05$
B	p_{BA}	$= 0.10$	p_{BB}	$= 0.80$	p_{BD}	$= 0.10$
D	p_{DA}	$= 0.00$	p_{DB}	$= 0.00$	p_{DD}	$= 1.00$

One-year rating transition probabilities

Standard and Poor's 1981-1991, 'no rating' eliminated, in percentages

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	89.10	9.63	0.78	0.19	0.30	0	0	0
AA	0.86	90.10	7.47	0.99	0.29	0.29	0	0
A	0.09	2.91	88.94	6.49	1.01	0.45	0	0.09
BBB	0.06	0.43	6.56	84.27	6.44	1.60	0.18	0.45
BB	0.04	0.22	0.79	7.19	77.64	10.43	1.27	2.41
B	0	0.19	0.31	0.66	5.17	82.46	4.35	6.85
CCC	0	0	1.16	1.16	2.03	7.54	64.93	23.19
D	0	0	0	0	0	0	0	100

Historical Default Probabilities vs. Credit Spreads

- The defaultable bond prices of the model do not agree with observed bond prices
- Credit spreads are too low. (Particularly for higher quality debt.)
- Reasons: Risk Premia, liquidity premia
- Credit spreads within rating classes are stochastic.
- Adjustments to the historical transition matrix:
  - Risk Premia
  - Stochasticity
  - Consistency: Monotonicity

Advantages

- ✔ Appropriate model for rating transitions.
- ✔ Much data available.
- ✔ Data is from independent analysis.
- ✔ (In simplest version) relatively straightforward.
- ✔ Can be adapted to bank-internal risk-scoring methods: Pricing and analyzing credit risk of a loan portfolio.

Disadvantages

based in the nature of the data:

- ✘ historical data (backwards-looking)
- ✘ Real world ratings adjustments happen with delay.
- ✘ Rating agencies may define defaults differently.
- ✘ Historical default probabilities do not justify spreads observed in the market. Need risk premium adjustment to fit model.
- ✘ Transition probabilities only depend on current rating (not on history). No 'ratings momentum' possible in the model.

- ✘ Data sometimes based on only very few transitions.
- ✘ Mostly US corporates.
- ✘ Spreads constant within same rating class.
- ✘ Full model of spreads computationally very complex:
  Need at least one Brownian motion for each rating class.
- ✘ For stochastic spreads deduction of generating matrix can become very complicated.

Fields of Application

- Pricing and investment tool if no other information available.
- A simpler alternative to modelling the full price dynamics of corporate bonds.
- Internal risk-scoring models.
- Derivatives that condition on rating transitions.
- Not for pure credit spread derivatives.

Reduced Form Models: Why Poisson Processes?

Ultimate goal:
A mathematical model of defaults that is realistic and tractable and useful for pricing and hedging.

Defaults are

- sudden, usually unexpected
- rare (hopefully :-)
- cause large, *discontinuous* price changes.

Require from the mathematical model the same properties.

Furthermore: The probability of default in a short time interval is approximately **proportional** to the length of the interval.

Advantages

- ✔ Credit spreads are directly modelled: the intensity of the default process *is* the credit spread.
- ✔ Can be fitted to observed credit spreads.
- ✔ Realistic credit spreads through discontinuous dynamics.
- ✔ Suitable for the pricing of credit derivatives.

Disadvantages

- ✘ Little explanatory value: credit spreads are inputs, not outputs.
- ✘ Modelling done under martingale measures is not suitable for VaR analysis.
- ✘ Default correlation is potentially difficult to model. (Though this problem is essentially solved — see Schönbucher/Schubert (2001) and Rogge/Schönbucher (2003).)

References

- **Black, F. and J. Cox** (1976). Valuing Corporate Securities: Some Effects of Bond Indenture Provisions. *Journal of Finance*, pages 351-367.
- **Black, F. and M. Scholes** (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81:637-54.
- **Briys, E. and F. de Varenne** (1997). Valuing Risky Fixed Rate Debt: An Extension. *Journal of Financial and Quantitative Analysis*, 32(2):239-248.
- **Crosbie, P.J.** (1999). Using Equity Price Information to Measure Default Risk. In: Francis, J.C., J.A. Frost and J.G. Whittaker, *Handbook of Credit Derivatives*, McGraw–Hill, New York.
- **Duffie, D. and K. J. Singleton** (1999). Modeling Term Structures of Defaultable Bonds. *The Review of Financial Studies*, 12(4):687-720.
- **Geske, R.** (1977). The Valuation of Corporate Liabilities as Compound Options. *Journal of Financial and Quantitative Analysis*, 541–552.
- **Jarrow, R.A., D. Lando and S.M. Turnbull** (1997). A Markov Model for the Term Structure of Credit Risk Spreads. *The Review of Financial Studies*, 10(2):481-523.
- **Jarrow, R.A. and S.M. Turnbull** (1995). Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*, 50:53-85.
- **Longstaff, F.A. and E.S. Schwartz** (1995). The Pricing of Credit Risk Derivatives. *Journal of Fixed Income*, 5(1):6-14.
- **Longstaff, F.A. and E.S. Schwartz** (1995). A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *Journal of Finance*, 50(3):789-819.
- **Merton, R.C.** (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, 29:449-470.
- **Rogge, E. and P.J. Schönbucher** (2003). Modelling Dynamic Portfolio Credit Risk. Working paper.
- **Schönbucher, P.J.** (2000). The Pricing of Credit Risk and Credit Derivatives. Unpublished manuscript, Department of Statistics, University of Bonn, Germany.
- **Schönbucher, P.J.** (2003). *Credit Derivatives Pricing Models,* John Wiley & Sons, Chichester, New York, Weinheim, Brisbane, Singapore and Toronto.
- **Schönbucher, P.J. and D. Schubert** (2001). Copula–Dependent Default Risk in Intensity Models. Working Paper, Department of Statistics, University of Bonn, Germany.
- **Zhou, C.** (1997). A Jump–Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities. Working paper, Federal Reserve Board, Washington, DC.