

Fair Pricing of Energy Derivatives

A Comparative Study

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Energy Risk Management - EnergyAustralia

April 30th, 2004

Abstract

The Australian National Electricity Market (NEM) is a highly volatile market in which price setting is driven by the law of supply and demand. The volatile nature of this market presents significant risk to market participants and as such stabilisation of price has implications for the prevention of financial distress.

To alleviate the risk associated with this volatility, over the counter - and eventually exchange traded - derivatives allow hedge positions to be established, thus resulting in a less volatile hedged exposure to the underlying. With the importance of derivative trading for market participants, consistent methods of derivative pricing become important.

This paper is a comparative study of two existing quantitative approaches for pricing energy derivatives, namely: a cash-flow at risk model based on Monte Carlo simulation of the underlying spot price and the SDE approach based on no-arbitrage theory utilising convenience yields and cost of carry to address the inability to store electricity.

The paper provides a presentation of the underlying theory of the two approaches with an empirical comparative study, contrasting calibration methods to quoted brokerage premiums and statistical methods of estimation from historical data. Advantages and disadvantages of both methods are discussed in light of the results obtained.

1 Motivation and Objectives

Defining the fair value of derivatives is not a simple task. In insurance, a fair value of a contingent claim is calculated by taking expectations of the claim payoff with addition of the claim variance or standard deviation to account for risk. In some other cases, when utility functions are used, the fair value is defined to be the certainty equivalent of the claim. The certainty equivalent is the amount which, when received by certainty, is regarded as good as taking the

*The author acknowledges support from Peter Murray and Tim O'grady. Discussion with Phil Moody was very useful and empirical work of Brett Gray is greatly appreciated.

risk. In financial markets, the fair value of an option is the expectation of the discounted payoff, where the expectation is taken under a different probability measure. When the underlying risk is assumed to be log-normally distributed, Black-Scholes like formulae for options can be obtained. It should be stressed that, in capital markets, many ad-hoc methods were used before the Black and Scholes methodology gained the consensus among different market participants. And even though it is widely used, it is well known that its assumptions are not realistic and it cannot be applied to pricing all types of options.

With the liberalisation of the Australian electricity industry, electricity prices are determined through a process of supply and demand and as such are floating in nature. The underlying spot price of NSW electricity has been observed to be highly volatile and at times of high demand, extreme price events occur representing a significant risk to market participants. As such, market participants have the need to hedge the risk associated with the volatile nature of the underlying and this is done through the trade of derivative contracts as similar to those found in the more typical capital markets. Such derivative trading could be considered to have indirect effects relating to the continuity of supply and the stability of the industry. If any market participant was to enter financial distress due to unhedged exposure, supply could potentially be affected.

With the importance of derivative trading there arises the need to determine appropriate derivative pricing techniques that are suitable to the structure of this market and existing methods adopted in the capital markets should be explored for this purpose. Among different alternatives, two approaches stand out:

- The cash flow at risk approach, based on historical price distribution. An option price is computed by taking the difference payments evaluated at each path of the Monte Carlo simulation and then computing a quantile to reflect the price that the market is trading at.
- The Stochastic Differential Equations (SDE) approach where the underlying dynamics are modeled via mean-reverting diffusion process, and then closed form solutions -akin Black and Scholes- for option prices can be obtained

Both approaches are genuine and have their strengths and weaknesses. The aim of this paper is to explore, and compare these two methodologies from theoretical and empirical aspects.

Furthermore, with the proposal for Australian electricity participants to adopt the IAS 39 international accounting standard currently adopted internationally (see <http://www.iasplus.com/standard/ias39.htm>), consistent valuation of energy derivatives is required here. The standard includes two possible methods for determining electricity derivative premia, an approach based on the assessment of market quotes and the other based on discounted cash-flow analysis. In studying the two methods above, we explore each of these options, adopting the SDE approach to calibrate to quoted premia and the cash-flow at risk approach as the basis for the discounted cash-flow analysis.

The paper is organised as the following. Section two details the idea and the mathematics behind the cashflow at risk based on historical price distribution. It explains the long range dependence concept and its use in the ARIMAX time series model. Then it explores a way to model spikes in historical prices using extreme value theory and Hill (1975) estimation.

Section three considers the SDE approach. It presents two different models known as Schwartz single and two factor models. Closed-form solutions to option prices are given in this framework as well as the risk sensitivities to underlying market parameters ("Greeks") for the one factor model.

Section four provides a comparison between the two approaches where calibration to broker cap premia methodology is explained and discussion of results is given.

2 Cashflow at Risk Based on Historical Price Distribution

This section details the cash flow at risk methodology adopted and is intended as a more intuitive account of the process.

It is well known in the literature and from empirical investigation that electricity spot prices exhibit cycles, seasonality and autocorrelation. This data structure can be captured and used to forecast future spot prices. We also know that occasional price spikes may occur due to unusual high load, network transmission constraints and unscheduled outages of generation. These extreme values and their probability of occurrence can also be captured from historical data and used in the Monte Carlo simulation.

In financial markets, Monte Carlo methods are recognised as very flexible tools for simulating future time series with which to evaluate cost and risk of various contracts. Possible future values can be generated by randomly sampling the relevant distributions, maintaining given volatility characteristics. Monte Carlo methods are relatively well understood, and can model correlations. Additionally such methods can achieve better accuracy by using complex dependency on the path taken to date.

The cashflow at risk methodology adopted in this paper is based on a methodology originally developed at Katestone Analytic and is based on a time series analysis approach. The methodology uses Monte Carlo simulation in which future simulated paths retain the cyclic and seasonal effects of historic spot as well as the autocorrelation structure and extreme price events typically observed.

The approach initially involves the development of a time series model of the underlying, which is to be estimated from historical data. Figure (1) illustrates such a model. Initially the historical data is raised to the power of 0.25 (a fourth root). This has the effect of dampening the extreme occurrences observed historically and makes the data more able to be analysed by other time series methods. Following this, the daily, weekly and yearly cyclic effects present in the

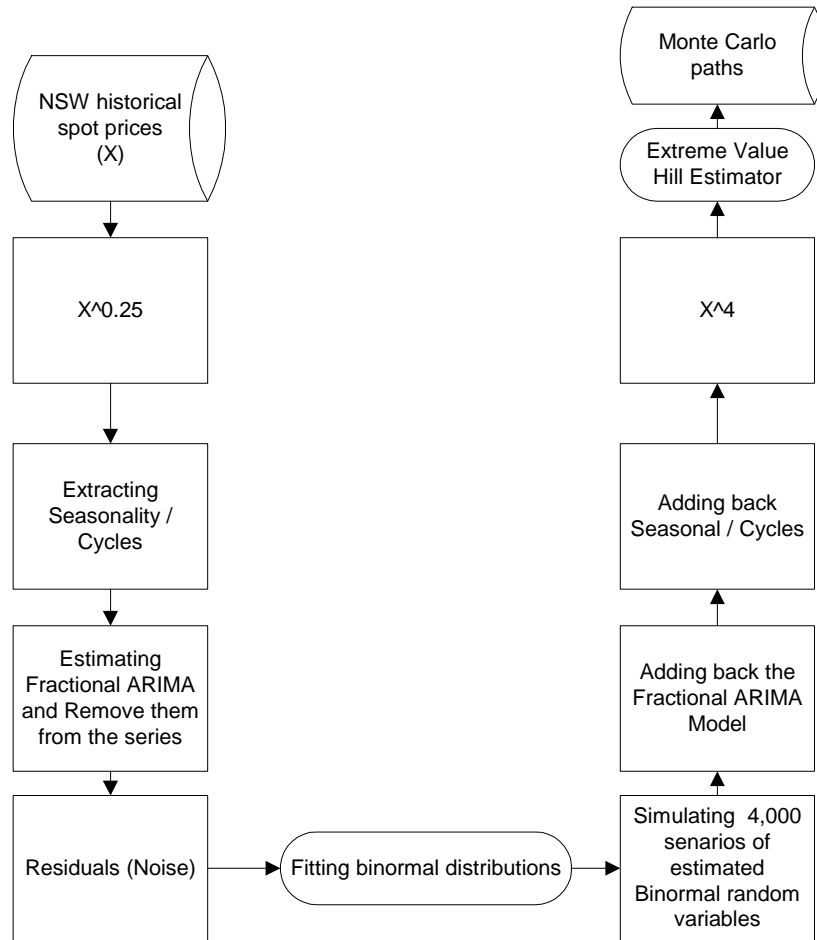


Figure 1: Univariate model used in generating Monte Carlo paths combined with extreme value theory

historical data are estimated. This allows the seasonal and peak/off-peak effects in the underlying series to be represented mathematically in a manner in which a Monte-Carlo method can incorporate these cyclic effects into the simulation process. Once this structure is estimated, it is removed from the historic data so that the series remaining is a representation of the historic underlying (to fourth root) that has been deseasonalised and had peak/off-peak effects removed.

A key feature of the simulation of sample paths for electricity market variables is the maintenance of observed serial correlation structure in the series. The fractional ARIMA time-series methodology is used to estimate and remove the temporal correlations and long memory from such variables. After this process, the final residuals of the analysis, after the removal of the cyclic and the fractional ARIMA estimation should represent values that are uncorrelated in time and have close to bi-normal distributions.

In order to generate a simulated path, the simulation process proceeds by repeated sampling from distributions estimated from the residuals of the above analysis, followed by a reconstruction of the full time-series by adding the estimated fractional ARIMA structure and cyclic information and inverting the fourth root operation initially adopted.

The distribution estimated from the final residuals is fitted to a mixture of Normal distributions and allows the proper treatment of longer, fatter tails in price distributions due to the physical constraints of the electricity market. Following the construction of such a simulated path, the Hill estimator is used to estimate the jump size and frequency from historical data and incorporate the extreme price events often observed in this industry.

Having generated a large number of representative price paths, the expected contract payouts for a given portfolio can be evaluated. Option prices and cash-flow at risk measures can then be evaluated from averages or higher quantiles and distributional parameters evaluated over the family of possible time series. Exotic options are readily treated by the standard decomposition into a linear combination of contract legs, where legs can be either cap, floor, swap or binary options.

2.1 Historical Prices Structure

2.1.1 Long Range Dependence

Karen Lunney (1995) observes: "Most environmental data can be described qualitatively as having long serial correlations in the time domain, flicker noise in the frequency domain and space filling capacity of the graphical representations of the data, also called fractal characteristics. Another common characteristic can be described as intermittency which can be defined as sudden bursts of high frequency behaviour".

Processes exhibiting these characteristics may have long range dependence. It is not surprising that we observe similar characteristics in electricity prices data; see figures (2), (3) and (4). This is because electricity prices mainly depend, among other factors, on electricity demand, which itself depends on

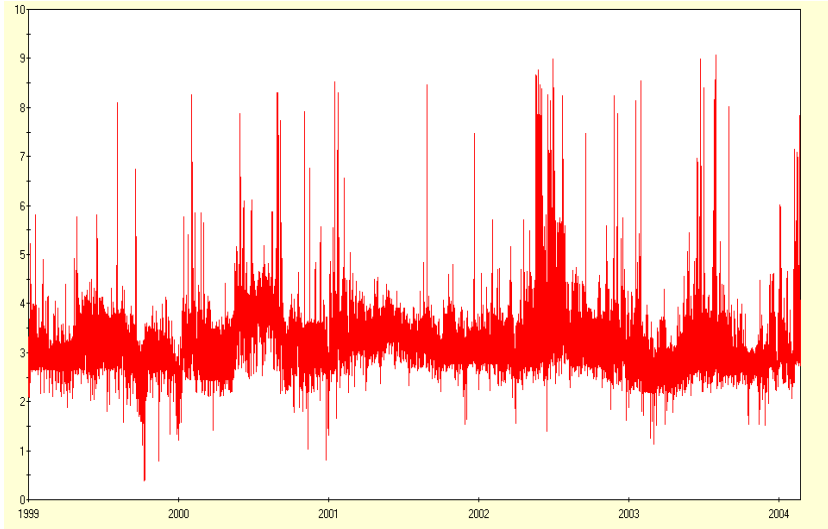


Figure 2: Time Plot of the Logarithm of Spot Prices since 1999

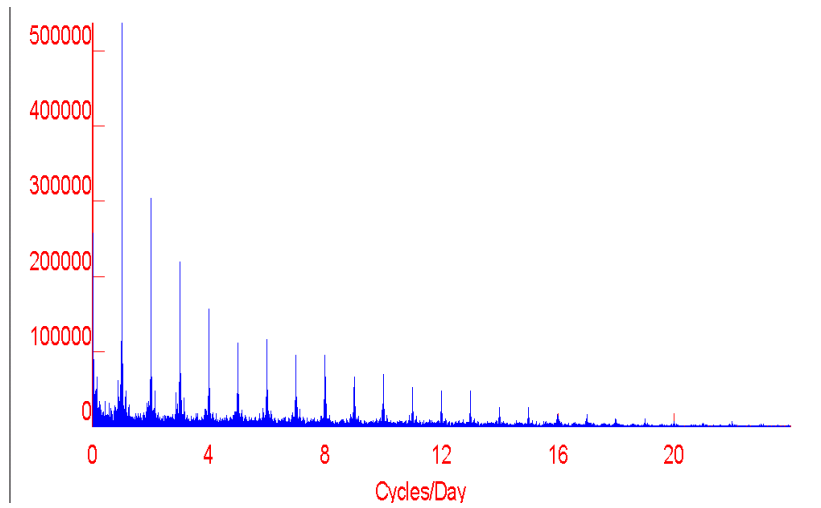


Figure 3: Fast Fourier Transform of Spot Prices

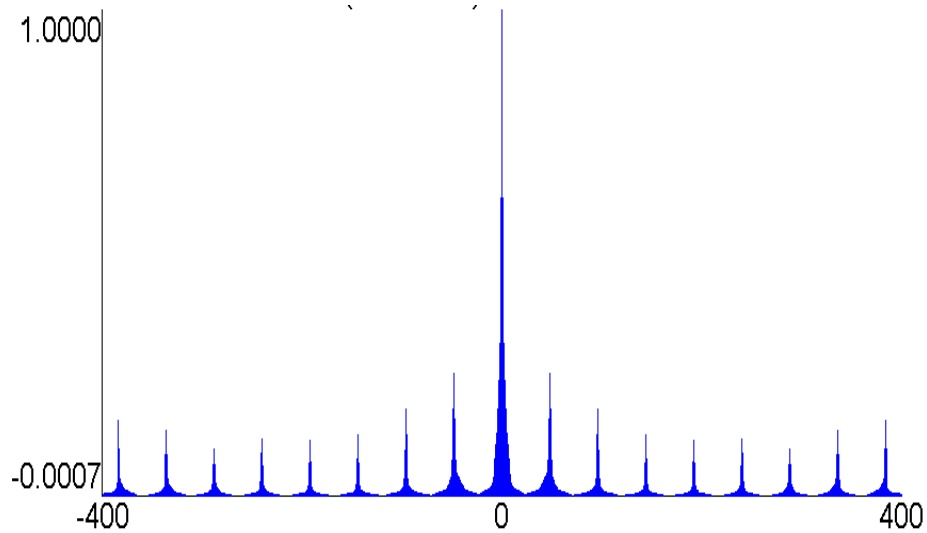


Figure 4: Autocorrelation of half-hourly spot prices

weather conditions. Many studies show that environmental measurement (such as rainfall, temperature, humidity, ozone concentrations, ...) have long temporal and spacial correlations. In order to better understand the idea of long range dependence, the field of critical phenomena is an area where long range dependence has physical interpretation. For example, a liquid may be in equilibrium with neighbouring molecules and the correlation relationship may be only weakly dependent. As the liquid approaches a critical temperature, however, where a phase transition to the gaseous phase occurs, the dependence structure changes with molecular correlation decaying very slowly, that is, long range dependence.

In the following section we present fractional ARIMA time series model which allows to capture the long range dependence.

2.1.2 Differencing and Standard ARIMA Models

The idea behind ARIMA Stationary models are an important class of stochastic models. This class of models assumes that the process remains in equilibrium about a constant mean level. Much development has been done to study the properties of such models. Now, if the process is non-stationary, -exhibiting explosive behaviour- it can be possible in some cases to transform it to a stationary process in order to use existing tools to study its properties, then backing out the non-stationary model parameters. The Autoregressive Moving Average models are an example of stationary time series.

ARIMA time series model, developed by Box and Jenkins are an extension

to the ARMA models using differencing¹. This method allows for the series to be differenced if necessary in order to obtain stationarity, and then the resulting series may be treated as an ordinary ARMA model. The I stands for the word integrating meaning summing, the inverse of the differencing operation. So, integrating an ARIMA process will yield an ARMA process.

Mathematical development We will now present a formulation of the *ARIMA* model based on Box *et.al.* (1994). A sequence $\{z_t\}_{t \geq 1}$ is said to be governed by the autoregressive moving average model *ARMA*(p, q) of orders p and q if it satisfies the following equation

$$z_t = (\varphi_1 z_{t-1} + \dots + \varphi_p z_{t-p}) + (\theta_0 e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots + \theta_q e_{t-q})$$

where (φ_i) and (θ_j) with $1 \leq i \leq p$ and $1 \leq j \leq q$ are constants and $\{e_t\}_{t \geq 1}$ is white noise. If we define the the lag operator B by $Bx_t = x_{t-1}$, hence, $B^m x_t = x_{t-m}$, for any integer m , then the above equation can be written as

$$(1 - \varphi_1 B - \dots - \varphi_p B^p)z_t = (\theta_0 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q)e_t$$

or

$$\varphi(B)z_t = \theta(B)e_t \tag{1}$$

where $\varphi(B)$ and $\theta(B)$ are polynomials in the lag operator B , of degrees p and q , respectively. An *ARMA* process is stationary if all the roots of $\varphi(B) = 0$ lie outside the unit circle², and is non-stationary if some of the roots lie inside the unit circle.

The third case which happens to be interesting is that for which the roots of $\varphi(B) = 0$ lie on the unit circle. It turns out that the resulting models are of great value in representing non-stationary time series.

Suppose that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle. Then the model (1) can be expressed in the form

$$\phi(B)(1 - B)^d z_t = \theta(B)e_t \tag{2}$$

where $\phi(B)$ is stationary autoregressive operator. If we denote the differencing operator ∇ , such that $\nabla x_t = x_t - x_{t-1} = (1 - B)x_t$, then the model (2) can be re-written as

$$\phi(B)w_t = \theta(B)e_t \tag{3}$$

$$\text{and } w_t = \nabla^d z_t \tag{4}$$

Thus we see that the model corresponds to assuming that the d th difference of the series can be represented by a stationary ARMA process.

¹Differencing a time series is transforming it to another time series where each element is the difference between two consecutive elements of the original time series

²if B^* is the solution to $\varphi(B) = 0$ then $|B^*| > 1$

For $d \geq 1$, the process z can be deduced from equation (4) to give

$$z_t = S^d w_t \tag{5}$$

where S is the infinite summation operator defined by

$$\begin{aligned} Sx_t &= \frac{1}{1-B}x_t \\ &= (1 + B + B^2 + B^3 + \dots)x_t \end{aligned}$$

Equation (5) implies that the process (2) can be obtained by summing or "integrating" the stationary process (3) d times. Hence the name autoregressive integrated moving average (ARIMA) process.

Now, the parameter d need not be integer. Indeed, it is always possible to write

$$\frac{1}{(1-B)^d} = \alpha_0 + \alpha_1 B + \alpha_2 B^2 + \dots$$

for example

$$\frac{1}{(1-B)^{0.5}} = 1 + \frac{1}{2}B + \frac{3}{8}B^2 + \frac{5}{16}B^3 + \dots$$

In the case when $0 \leq d \leq 1$, the model is called fractional ARIMA. In section (2.1.3), we present an algorithm for estimating d from the data.

2.1.3 Fractional ARIMA Models

The standard ARIMA models as presented in the previous section use an integer differencing parameter. However, this parameter can be any real number between 0 and 1, given the argument at the end of the previous section. In this case, they are called fractional ARIMA models.

In the previous section we analysed the properties of ARMA models in the time domain, that is the properties of $\{z_t\}_{t \geq 1}$ where t represents time. An equivalent analysis of the time series can be performed in the frequency domain.

The idea behind going back to frequency domain An alternative way of analysing a time series is to assume that it is made up of sine and cosine waves with different frequencies. A device that uses this idea is the periodogram. Suppose that the number of observations in the time series, $N = 2q + 1$ is odd. We can fit the Fourier series model

$$z_t = \alpha_0 + \sum_{i=1}^q (\alpha_i c_{it} + \beta_i s_{it}) + e_t$$

where $c_{it} = \cos(2\pi f_i t)$, $s_{it} = \sin(2\pi f_i t)$, and $f_i = \frac{i}{N}$ is the i th harmonic of the fundamental frequency $\frac{1}{N}$, the least squares estimates of the coefficients α_0 and

(α_i, β_i) will be

$$\begin{aligned}\widehat{\alpha}_0 &= \bar{z} \\ \widehat{\alpha}_i &= \frac{2}{N} \sum_{t=1}^N z_t c_{it} \\ \widehat{\beta}_i &= \frac{2}{N} \sum_{t=1}^N z_t s_{it}\end{aligned}$$

for $i = 1, 2, \dots, q$

The periodogram. consists of the $q = \frac{(N-1)}{2}$ values

$$I(f_i) = \frac{N}{2} \left(\widehat{\alpha}_i^2 + \widehat{\beta}_i^2 \right)$$

where $I(f_i)$ is called the intensity at frequency f_i .

The periodogram is an appropriate tool for analysing time series made up of mixtures of sine and cosine waves, at fixed frequencies buried in noise. However, stationary time series of the ARIMA kind are characterised by random changes in frequency, amplitude and phase. For this type of series the periodogram fluctuates widely, hence mean values of the intensity functions are considered. The definition of the periodogram is generalised to

$$I(\omega) = \frac{N}{2} \left(\widehat{\alpha}_\omega^2 + \widehat{\beta}_\omega^2 \right)$$

where ω is the frequency that can vary continuously in the range $[0, 0.5]$ and does not need to be a multiple of $\frac{1}{N}$. We also define the power spectrum $p(f)$ as

$$p(\omega) = \lim_{N \rightarrow \infty} \mathbb{E} [I(\omega)]$$

This introduces the spectral density of a time series as:

$$f(\omega) = \frac{p(\omega)}{\sigma_z^2}$$

where σ_z^2 is the variance of the process z assumed to be constant.

Estimation of the long range dependence parameter In this section, based on the work of Lunney (1996), we present the mathematical derivation and an algorithm for estimating the differencing parameter d introduced in section (2.1.2).

The estimation of the differencing parameter d as well as the terms of the Autoregressive part of the ARIMA is easier to be performed in the frequency domain rather than the time domain. It can be shown (see Box *et al.* (1994)) that the spectral density of an ARIMA process $\{z_t\}_{t \geq 1}$ has the following form

$$f(\omega, \theta) = \frac{\sigma^2}{2\pi} |1 - e^{i\omega}|^{-2d} |\phi_h(e^{i\omega})|^{-2}, 0 < d < \frac{1}{2}, \omega \in (-\pi, \pi) \quad (6)$$

where $\theta \in \Xi$, the vector of all parameters on the right hand side of equation (6), and Ξ is a compact subset of \mathbb{R}^n , n being the number of elements of θ , and

$$\phi_h(e^{i\omega}) = 1 - \phi_1 e^{i\omega} - \phi_2 e^{i2\omega} - \dots - \phi_h e^{ih\omega}$$

with $\phi_h(z) \neq 0$ for $|z| < 1$. The factor $|1 - e^{i\omega}|^{-2d}$ in (6) corresponds to d th order differencing.

The estimation of the vector of parameters θ can be obtained by minimising

$$\min_{\theta} \int_{-\pi}^{\pi} \left[\log f(\omega, \theta) + \frac{I_T(\omega)}{f(\omega, \theta)} \right] d\omega \quad (7)$$

where $I_T(\omega)$ is the unbiased periodogram based on the sample $\{z_1, \dots, z_T\}$. This yields the asymptotic maximum likelihood estimator of θ . It can be shown from Lunney (1995) that problem (7) is equivalent to

$$\min_{\theta} \int_{-\pi}^{\pi} \left[\frac{I_T(\omega)}{f_o(\omega)} \right] d\omega$$

where

$$f_o(\omega) = |1 - e^{i\omega}|^{-2d} |\phi_h(e^{i\omega})|^{-2}$$

Let us define

$$\frac{I_T(\omega)}{f_o(\omega)} = \epsilon_T(\omega) \quad (8)$$

We have $\mathbb{E}[\epsilon_T(\omega)] = 1$ since $I_T(\omega)$ is unbiased estimator of $f_o(\omega)$. Also, as shown by Yajima (1989), $\{\epsilon_T(\omega_j)\}$ is an asymptotically independent sequence under some conditions which are satisfied by model (6) here for the Fourier frequencies. Equation (8) yields

$$\log I_T(\omega) = \log f_o(\omega) + \log \epsilon_T(\omega)$$

At the Fourier frequencies, this equation becomes

$$\log \frac{I_T(\omega_j)}{\phi_h(\omega_j)} = -d \log \left(2 \sin \frac{\omega_j}{2} \right)^2 + \log \epsilon_T(\omega_j) \quad (9)$$

Now, we can estimate the differencing parameter d using least squares applied to the regression (9), so that

$$\hat{d} = - \frac{\sum_{0 < \omega_j < \pi/3} \log \frac{I_T(\omega_j)}{\phi_h(\omega_j)} \log \left(2 \sin \frac{\omega_j}{2} \right)^2}{\sum_{0 < \omega_j < \pi/3} \left(\log \left(2 \sin \frac{\omega_j}{2} \right)^2 \right)^2} \quad (10)$$

Yet to be estimated the unknown parameters of ϕ_h . So we re-write equation (9) as

$$\log I_T(\omega_j) = \log \phi_h(0) - d \log \left(2 \sin \frac{\omega_j}{2} \right)^2 + \log \frac{\phi_h(\omega_j)}{\phi_h(0)} + \log \epsilon_T(\omega_j) \quad (11)$$

At low frequencies, i.e. ω is near zero, the term $\log \frac{\phi_h(\omega_j)}{\phi_h(0)}$ is negligible relative to the other terms on the right hand side of (11). Therefore, d can roughly be estimated by applying least squares to the regression (11). This estimated value of d is an approximation that will be considered as an initial value of d .

Now, the order and parameters of ϕ_h can now be estimated by taking the Discrete Fourier Transfer (DFT) of

$$I_s(\omega_j) = \left(2 \sin \frac{\omega_j}{2}\right)^{2\hat{d}_0} I_T(\omega_j)$$

This will yield estimates of the covariance function of $w_t = (1 - B)^d z_t$. An $AR(h)$ model can now be fitted to these covariances using the Durbin-Levinson recursion. BIC can be used to select the model order, where

$$BIC = \log \sigma_{h,d}^2 + h \frac{\log T}{T}$$

$\sigma_{h,d}^2$ being the minimum prediction error variance for the data given d and h . Now, given h , the parameters ϕ_j , $j = 1, \dots, h$, of ϕ_h can be estimated using maximum likelihood.

At this step, given the estimated values of h and parameters, ϕ_i , we can use equation (10) to compute an unbiased estimator for d . This estimator can in turn be used in the AR fitting step to gain better estimates for h and ϕ_i . This procedure can be iterated until convergence to accurate values of d , h and ϕ_j , $j = 1, \dots, h$ is obtained.

2.2 Extreme Values Estimation

The normal distribution has a normal tail, which means that values beyond three standard deviation have less than 0.27% of occurrence. In order to model electricity prices where extreme values have higher probability of occurrence, fat tail distributions should be considered. There are families of distributions which have this property, such as elliptical distributions and extreme value distributions, i.e. Gumbel, Fréchet or Weibull (see Kotz and Nadarajah 2000). Once an extreme value distribution is chosen, some of its parameters indicate the fatness of the tail.

The Hill estimator essentially looks at the data and estimates the fatness parameter so that the resulting distribution has the appropriate shape.

2.2.1 Mathematical development

As pointed earlier, electricity prices are heavy-tailed. One of the issues in the study of long-tailed probability distributions is the estimation of the tail index α , which measure the heaviness the tail of the distribution. Let X_1, X_2, \dots, X_n be an independent and identically distributed random sample from a long-tailed distribution F such that, for sufficiently large x , the distribution F is assumed to have the algebraic functional form. That is, $1 - F(x) \sim Cx^{-\alpha}$ for $x \geq D$,

where α is the (upper) tail index, D is the threshold above which the assumed algebraic form is valid, and C is a normalising constant. Define $X^{(1)} \geq X^{(2)} \geq \dots \geq X^{(n)}$ as the descending order statistics from the distribution F . A tail index estimator derived from maximum likelihood considerations and known as the Hill estimator (Hill 1975), is defined as

$$\alpha_H^{(r)} = \frac{r + 1}{\sum_{i=1}^r i \ln \left(\frac{x^{(i)}}{x^{(i+1)}} \right)},$$

where $r + 1$ is the number of observations above the threshold D . The implementation of the Hill estimator, however, requires a choice of the number of extreme order statistics, r , from a sample of size n , where $2 < r + 1 \leq n$, or equivalently the threshold D .

There are methods which allow estimation of the threshold D from data (see Hsieh 1999). In our analysis, we take D as an input. Indicative values for peak season $D = \$100$ and for off-peak $D = \$70$.

Section (4) below presents simulations using the above cash-flow at risk methodology and compares results to the SDE approach detailed below.

3 Stochastic Differential Equation Approach

The SDE approach is another paradigm in modelling electricity prices. It has the advantage of representing the dynamics of the underlying forward or spot prices with a stochastic differential equation and using the stochastic calculus in deriving closed form solutions to option prices written on this underlying. In many electricity markets a mean-reverting and jump-diffusion process is popularly used in modelling the electricity spot price; see for example Deng (2000), Johnson and Barz (1999) and Clewlow, Strickland and Kaminski (2001). Nevertheless, there are two important challenges in pricing electricity derivatives. The first challenge is accurate parameters estimation of the mean-reverting jump diffusion stochastic process. The second is to derive a pricing measure under which option prices equal the mean of discounted payoffs.

It is clear that the more factors the model has, the more characteristics of electricity it can capture, i.e. mean reversion, seasonal effects, price dependent volatility, occasional price spikes... However, this means more parameters to calibrate.

Given the lack of liquidity in the electricity market, the calibration of multi-factor models is not stable. Let us consider the one and two factor models to study the stability of the calibration and the consistency of pricing. We base our presentation of the models on Clewlow and Strickland (2000).

3.1 One Factor Model

The single factor model introduced by Schwartz (1997) assumes that the spot price follows a mean-reverting stochastic process.

$$\frac{dS_t}{S_t} = \alpha [\mu - \ln(S_t)] dt + \sigma dW_t \quad (12)$$

where α is the mean reversion rate (the speed of adjustment of the spot price back towards its long term level μ) and σ is the spot price volatility. Assuming a constant short term interest rate over the period $[t, T]$, European call option price is given by:

$$c(t, K, T) = e^{-r(T-t)} [F(t, T)N(h) - KN(h - \sqrt{w})]$$

where

$$h = \frac{\ln\left(\frac{F(t, T)}{K}\right) + \frac{1}{2}w}{\sqrt{w}}, \quad w = \frac{\sigma^2}{2\alpha} [1 - e^{-2\alpha(T-t)}]$$

$N(\cdot)$ is the standard normal cumulative distribution function, and

$$F(t, T) = \exp\left[e^{-\alpha(T-t)} \ln(S_t) + \left(1 - e^{-\alpha(T-t)}\right) \left(\mu - \frac{\sigma^2}{2\alpha}\right) + \frac{\sigma^2}{4\alpha} \left(1 - e^{-2\alpha(T-t)}\right)\right]$$

Here r is the continuously compounded short term interest rate. This model needs three parameters to be calibrated. We shall detail the calibration exercise in section 3.4. It is worth noting that one does not need to implement the dynamics of the underlying as given in equation (12) in order to price options on this underlying. The closed form of the call option $c(t, K, T)$ is readily computed without simulation.

3.2 Two Factor Model:

3.2.1 Convenience yield

The convenience yield measures the advantage of holding the commodity minus the cost of storage. In electricity markets, plants seek to minimise their cost of production by avoiding the cost of shutting down and restarting the plant due to high prices or lack of available supplies. Hence the concept of convenience yield can be applicable when the source of electricity is fuel or coal.

3.2.2 The model

Although simple to implement, the one factor model has very simple volatility structure which goes to zero with increasing maturities. Schwartz (1997) introduce a two factor model where the first factor is the underlying spot price and the second factor is the convenience yield. The first factor is the spot price which is assumed to follow a diffusion process

$$dS_t = (r - \delta_t)S_t dt + \sigma S_t dW_t$$

where r is the short term interest rate and δ_t is the convenience yield at time t , σ is spot price volatility and W is the Brownian motion.

The second factor is the instantaneous convenience yield of spot energy and has the following mean-reverting process:

$$d\delta_t = \alpha_\delta(\bar{\delta} - \delta_t)dt + \sigma_\delta dZ_t$$

where α_δ and $\bar{\delta}$ represent the speed of adjustment and long term mean of the convenience yield, σ_δ is the convenience yield volatility and Z_t is the Brownian motion. The two Brownian motions used in the spot price and the convenience yield dynamics, W_t and Z_t are correlated with correlation coefficient $\rho_{S\delta}$, i.e. $dW_t dZ_t = \rho_{S\delta} dt$.

The forward energy prices can be derived (Schwartz (1997))

$$F(t, s) = S_t \exp \left[-\delta_t \frac{1 - e^{-\alpha_\delta(s-t)}}{\alpha_\delta} + A(t, s) \right]$$

where

$$\begin{aligned} A(t, s) = & \left(r - \bar{\delta} + \frac{1}{2} \frac{\sigma_\delta^2}{\alpha_\delta^2} - \frac{\sigma\sigma_\delta\rho_{S\delta}}{\alpha_\delta} \right) (s-t) + \frac{1}{4} \frac{\sigma_\delta^2}{\alpha_\delta^3} \frac{1 - e^{-2\alpha_\delta(s-t)}}{\alpha_\delta^3} + \\ & \left(\bar{\delta}\alpha_\delta + \sigma\sigma_\delta\rho_{S\delta} - \frac{\sigma_\delta^2}{\alpha_\delta} \right) \frac{1 - e^{-\alpha_\delta(s-t)}}{\alpha_\delta^2} \end{aligned}$$

The price of a European call option, maturing at time T , with strike price K on the spot price is given by:

$$c(t, K, T) = e^{-r(T-t)} [F(t, T)N(h) - KN(h - \sqrt{w})]$$

where

$$h = \frac{\ln [F(t, T)/K] + \frac{1}{2}w}{\sqrt{w}}$$

$$\begin{aligned} w = & \sigma^2(T-t) - \frac{2\sigma\sigma_\delta\rho_{S\delta}}{\alpha_\delta} \left((T-t) - \frac{(1 - e^{-\alpha_\delta(T-t)})}{\alpha_\delta} \right) + \\ & \frac{\sigma_\delta^2}{\alpha_\delta^2} \left((T-t) - \frac{2}{\alpha_\delta} (1 - e^{-\alpha_\delta(T-t)}) + \frac{1}{2\alpha_\delta} (1 - e^{-2\alpha_\delta(T-t)}) \right) \end{aligned}$$

3.3 Pricing Caps

Energy price caps limit the floating price of energy the holder will pay on a predetermined set of dates $T + i\Delta T$; $i = 1, \dots, N$ to a fixed cap level K . A cap is therefore a portfolio of standard European call options with its price given by

$$CAP(t, K, T, N, \Delta T) = \sum_{i=1}^N c(t, K, T + i\Delta T)$$

3.4 Calibration

In the calibration exercise, we use the observed market prices taken from broker screens to choose the model parameters so that the prices returned by the model coincide with the observed market prices. Let $C_{market,i}$, $i = 1, \dots, N$ denote the set of market prices and $C_{model,i}(\Theta)$ is the corresponding model prices set. Θ is the set of parameters of the option pricing model, for example, in Schwartz two factor model, $\Theta = [\sigma, \alpha_\delta, \bar{\delta}, \sigma_\delta, \rho]$. In practice, the number of the parameters in the model is less than the observed market prices. Therefore, the model prices will not exactly equal the observed ones. Hence, the procedure consists of finding the model parameters Θ for which the model prices are close to the market ones by minimising the sum of square of relative differences between the prices:

$$\underset{\Theta}{\text{minimise}} \left\{ \sum_{i=1}^N \left(\frac{C_{market,i} - C_{model,i}(\Theta)}{C_{market,i}} \right)^2 \right\}$$

3.5 Option Sensitivities: The "Greeks"

The derivative value (in mathematical sense) represents the sensitivity of the option to market variables and time. Some well known derivatives include delta, gamma, vega and theta. The delta measures the sensitivity of the option value change to the change in the spot prices.

$$\Delta_t = \frac{\partial C_t}{\partial S_t}$$

For a one dollar change in the spot price, for example, the delta of the option is the dollar change in the option value.

Gamma represents the change in the delta of an option when the underlying spot price changes by one dollar.

$$\Gamma_t = \frac{\partial \Delta_t}{\partial S_t} = \frac{\partial^2 C_t}{\partial S_t^2}$$

Vega risk represents the option value change due to unit change in the volatility:

$$V_{t,T} = \frac{\partial C_t}{\partial \sigma_{t,T}}$$

For a 1% change in the underlying volatility, there is $0.01 \times V_{t,T}$ dollar change in the value of the option.

As an option approaches its expiration, its value converges to its payoff (difference payment or intrinsic value). Therefore, options lose value over time due to this lack of optionality. This time decay of an option value is called the theta, Θ , and is defined as

$$\Theta = \frac{\partial C_t}{\partial t}$$

This value can be used to amortize the premium paid for an option through time.

For caps, we derive the delta, the gamma and the vega of the option in the case of the one factor Schwartz model (section 3.1).

- The delta is

$$\Delta_t = \frac{e^{-(r+\alpha)(T-t)}}{S_t} \left[F(t, T) \left(N(h) + N'(h) \frac{1}{\sqrt{w}} \right) - KN'(h - \sqrt{w}) \frac{1}{\sqrt{w}} \right].$$

- The Gamma is

$$\Gamma_t = -\frac{\Delta_t}{S_t} + \frac{e^{-(r+\alpha)(T-t)}}{S_t} \left[\frac{F(t, T)}{S_t} e^{-\alpha(T-t)} \left(N(h) + N'(h) \frac{1}{\sqrt{w}} \right) + F(t, T) \left(\frac{\partial N(h)}{\partial S} + \frac{\partial N'(h)}{\partial S} \frac{1}{\sqrt{w}} \right) - K \frac{\partial N'(h - \sqrt{w})}{\partial S} \frac{1}{\sqrt{w}} \right]$$

where

$$\begin{aligned} \frac{\partial N(h)}{\partial S} &= \frac{e^{-\alpha(T-t)} N'(h)}{S_t \sqrt{w}} \\ \frac{\partial N'(h)}{\partial S} &= \frac{e^{-\alpha(T-t)} N''(h)}{S_t \sqrt{w}} \\ \frac{\partial N'(h - \sqrt{w})}{\partial S} &= \frac{e^{-\alpha(T-t)} N''(h - \sqrt{w})}{S_t \sqrt{w}} \end{aligned}$$

- The Vega is

$$V_{t,T} = e^{-r(T-t)} \left[\frac{\partial F(t, T)}{\partial \sigma} N(h) + F(t, T) \frac{\partial N(h)}{\partial \sigma} - K \frac{\partial N(h - \sqrt{w})}{\partial \sigma} \right]$$

where

$$\begin{aligned} \frac{\partial F(t, T)}{\partial \sigma} &= F(t, T) \frac{\sigma}{\alpha} e^{-\alpha(T-t)} [1 - \cosh(\alpha(T-t))] \\ \frac{\partial N(h)}{\partial \sigma} &= \left[\begin{array}{l} -\frac{\sigma}{2\alpha} w^{-\frac{3}{2}} [1 - e^{-2\alpha(T-t)}] \ln \left(\frac{F(t, T)}{K} \right) + \\ w^{-\frac{1}{2}} \frac{\sigma}{\alpha} e^{-\alpha(T-t)} [1 - \cosh(\alpha(T-t))] + \\ \frac{\sigma}{4\alpha} w^{-\frac{1}{2}} [1 - e^{-2\alpha(T-t)}] \end{array} \right] \frac{1}{\sqrt{2\pi}} e^{-\frac{h^2}{2}} \\ \frac{\partial N(h - \sqrt{w})}{\partial \sigma} &= \left[\frac{\partial N(h)}{\partial \sigma} \frac{e^{\frac{h^2}{2}}}{\sqrt{2\pi}} - \frac{1}{2} w^{-\frac{1}{2}} \frac{\sigma}{\alpha} (1 - e^{-2\alpha(T-t)}) \right] \frac{1}{\sqrt{2\pi}} e^{-\frac{(h - \sqrt{w})^2}{2}} \end{aligned}$$

4 Empirical Study of the Two Approaches

We use 3 broker screens provided via Reuters to have quotes on caps and floors prices, Prebon Yamane Company (SPARK03), TFS Australia (TFSOPTS) and ICAP (ICAELOPT). On March 1st, 2004, there were 14 listed contracts. Table

Contract	Market Premium \$/MWH	Cashflow at Risk \$/MWH	Single Factor \$/MWH	Two Factor \$/MWH
NSW Q204 FLAT CAP 300	7.88	10.74	3.49	1.35
NSW APR-DEC 04 OFF PEAK CAP 300	2.35	3.85	3.39	3.22
NSW APR-DEC 04 FLAT CAP 300	6.25	4.68	3.39	3.26
NSW APR-DEC 04 FLAT CAP 100	6.80	5.86	8.39	4.88
NSW Q304 FLAT CAP 300	8.00	7.11	3.41	3.85
NSW FIN 04/05 FLAT CAP 300	7.63	3.93	3.28	4.04
NSW FIN 04/05 FLAT CAP 100	8.13	5.09	8.03	5.04
NSW Q404 FLAT CAP 300	2.75	1.17	3.28	4.56
NSW Q105 PEAK CAP 300	28.50	5.26	3.23	4.22
NSW Q105 FLAT CAP 300	12.50	4.12	3.23	4.22
NSW Q105 FLAT CAP 100	13.50	5.94	7.92	4.85
NSW CAL 05 FLAT CAP 300	7.58	3.79	3.18	3.18
NSW CAL 05 FLAT CAP 100	8.38	4.95	7.80	3.49
NSW CAL 05 OFF PEAK CAP 300	2.18	3.31	3.18	3.19

Table 1: Market vs. model implied premiums of brokered contracts

(1) presents the broker quotes with the valuations of the three models over the same set of contracts while table (2) presents absolute percentage errors of valuations and MAPEs (Mean Absolute Percentage Errors) for each of the models. We will now proceed to discuss these results in terms for each of the three models tested.

4.1 Cash flow at Risk Monte-Carlo Methodology

The cash flow at risk Monte-Carlo model was estimated with historical pool data from 1/1/1999 – 1/3/2004. A simulation was then run, generating 4000 simulated paths of future spot price and these paths were used to calculate the profit distribution of the difference payments incurred by holding each of the broker contracts. A model implied contract premium was then determined by adopting a given quantile (percentile) of the profit distribution and considering that quantile to be the premium (converted to a \$/MWH figure). The selection of the appropriate quantile to adopt provides an indication of the risk premium inherent in the contracts' premia. The quantile at a probability of 0.5 (i.e. the 50th percentile) could be considered a contract value in which no risk premium is present. As the quantile moves further up the profit distribution, the risk premium can be considered larger.

In order to determine the appropriate quantile of the profit distributions to adopt, the quantile's probability was optimised to the observed option premia such that the objective function presented in section (3.4) was minimised. The quantile's probability as optimised over the entire set of broker contracts, was the largest 0.99075 (i.e. the 99.075th percentile) of the profit distribution.

Contract Name	Cashflow at Risk	Single Factor	Two Factor
NSW Q204 FLAT CAP 300	36.34	55.70	82.82
NSW APR-DEC 04 OFF PEAK CAP 300	63.96	44.21	37.16
NSW APR-DEC 04 FLAT CAP 300	25.06	45.70	47.84
NSW APR-DEC 04 FLAT CAP 100	13.85	23.34	28.21
NSW Q304 FLAT CAP 300	11.14	57.31	51.92
NSW FIN 04/05 FLAT CAP 300	48.42	56.97	46.96
NSW FIN 04/05 FLAT CAP 100	37.38	1.19	37.98
NSW Q404 FLAT CAP 300	57.31	19.21	65.83
NSW Q105 PEAK CAP 300	81.53	88.66	85.20
NSW Q105 FLAT CAP 300	67.03	74.15	66.20
NSW Q105 FLAT CAP 100	55.99	41.35	64.06
NSW CAL 05 FLAT CAP 300	49.94	57.99	57.96
NSW CAL 05 FLAT CAP 100	40.90	6.90	58.28
NSW CAL 05 OFF PEAK CAP 300	52.00	46.32	46.56
Mean Absolute Percentage Error	45.77	44.21	55.50

Table 2: Absolute percentage error in model valuations and MAPE for each of the three models

This is implying an excessively large risk premium. Essentially the interpretation of this probability is that holding such a cap would only provide difference payments in excess of the contract's premium in 0.925% of cases. While this level of risk premium appears excessive, it can be explained by the methodology adopted of replicating historical price structure. As detailed above, the model for the underlying is estimated from historical data such that the paths generated would display similar structural and distributional properties to historical spot price. Table (3) shows the option premia quotes for flat \$300 caps of the different quarters and the difference payment that would result from holding such caps in history (i.e. the \$/MWH difference payment of a \$300 cap of the relevant quarter observed historically from 1/1/1999 – 1/3/2004). As can be seen from this figure, current broker quotes are significantly higher than the observed historical difference payment and as such, this data consistent with the calibrated model's risk premium when the assumption is adopted that the model volatility is estimated from historical data. Table (3) indicates that for a distribution of spot estimated from historical data, the quoted premia would be a high quantile of the contracts cost distribution implied by the model of the underlying.

Such a large observed risk premium would most likely be due to a market expectation that future volatility may be significantly higher than that observed historically as well as the potentially large losses that can occur from sold cap in a scenario of high volatility. In the SDE approach, as tested in this paper, the SDE representing the underlying process is calibrated to the quoted option premia. A potential benefit of the SDE approach in contrast to the cash flow at risk approach tested here is that higher expectations of future market volatility

Contract	Quoted Option	Difference Payment
	Premium (\$/MWH)	Observed Historically (\$/MWH)
NSW Q204 FLAT CAP 300	7.88	6.73
NSW Q304 FLAT CAP 300	8.00	3.84
NSW Q404 FLAT CAP 300	2.75	1.19
NSW Q105 FLAT CAP 300	12.50	3.94

Table 3: Option premium versus historic difference payment measured over historical spot data from 1999

Contract Subset	Profit Quantile Probability
All contracts	0.99075
\$100 Strike Caps	0.99975
\$300 Strike Caps	0.97975
Peak Caps	0.94650
OffPeak Caps	0.91000
Flat Caps	0.99750

Table 4: Calibrated quantile probability of the profit distribution when optimised on different subsets of the broker contracts. The degree above 0.5 is an indication of the risk premia for the contracts

can be quantified from quoted market premia. As such, the market premia will to some degree provide an indication of the expectation of the future underlying process. An elaboration of this point is discussed below.

This methodology can be used to determine risk premia by different classifications of the instruments. Table (4) shows optimised quantile probability of the profit distributions when calibrated to different subsets of the broker quoted contracts.

As would be expected, the risk premium for \$100 caps is greater than that for the \$300 caps. Additionally, the risk premium for Peak caps is higher than for off-peak caps. Surprisingly, the premium on flat caps is higher than that for peak or off-peak. This result may be due however to the specific duration of peak and off-peak caps in the data set and the lack of liquidity for these contracts.

The risk premia observed in table (4) provide some insight into the pricing results observed in table (1). As the pricing in table (1) is based on the quantile probability optimised with all contracts, the comparatively low risk premium on the off-peak caps means the model has overpriced these caps. The \$100 caps and the majority of the flat caps have been underpriced due to the higher risk premium here.

An important property to study for such a model is the stability of model implied prices over time. As such, the model was used to value the same set of contracts prices with an additional two days historical spot data. The model was re-estimated with the extra data, paths were regenerated and the same profit distribution quantile probability was adopted as calibrated from the initial esti-

Contract	Cashflow at Risk \$/MWH 1/1/03	Cashflow at Risk \$/MWH 3/1/04	Percentage change in contract value
NSW Q204 FLAT CAP 300	10.74	11.069	3.09
NSW APR-DEC 04 OFF PEAK CAP 300	3.85	3.969	3.01
NSW APR-DEC 04 FLAT CAP 300	4.68	4.814	2.78
NSW APR-DEC 04 FLAT CAP 100	5.86	5.97	1.91
NSW Q304 FLAT CAP 300	7.11	7.262	2.15
NSW FIN 04/05 FLAT CAP 300	3.93	3.879	-1.37
NSW FIN 04/05 FLAT CAP 100	5.09	5.036	-1.02
NSW Q404 FLAT CAP 300	1.17	1.026	-12.61
NSW Q105 PEAK CAP 300	5.26	5.222	-0.80
NSW Q105 FLAT CAP 300	4.12	4.295	4.22
NSW Q105 FLAT CAP 100	5.94	6.097	2.61
NSW CAL 05 FLAT CAP 300	3.79	3.799	0.18
NSW CAL 05 FLAT CAP 100	4.95	4.941	-0.18
NSW CAL 05 OFF PEAK CAP 300	3.31	3.362	1.69

Table 5: Cashflow at risk model valuation of contracts on the 1/1/2004 and the 3/1/2004

mation (i.e. 0.99075). Over such a short duration, if the methodology is robust for pricing, in the absence of major news entering the market a large difference in contract pricing should not be observed. Table (5) shows the original option prices from the model and the prices determined in the estimation including the additional two days data. As can be seen the percentage change in price is not prohibitive for the use of such a model. The larger change observed for the Q404 Flat \$300 Cap is due to its small valuation, as a result of the lack of extreme prices typically observed in quarter four. The contributing factors to the observed price changes include an effect of the different valuation dates on the present valuing of the profit distribution and an effect from the additional two days estimation data. Due to the time differential between the two valuations, we believe these effects would be negligible. As such the primary reason for the observed percentage change in contract values would be a lack of complete convergence to the theoretically implied profit distribution. Such an effect could be remedied through the generation of more paths or the incorporation of more efficient monte-Carlo random number generation techniques such as antithetic variables and pseudo-random numbers (see Putney 1999).

4.2 Stochastic Differential Equation Approach Results

The single factor model achieved the most accurate calibration to the broker data of all the models tested. The additional accuracy over the cash flow at risk model may be due to its process of calibrating all model parameters to the broker data rather than basing its estimation on historical spot data. This has the potential benefit that a model of the underlying is essentially determined

via a market consensus through the quoted premia, however issues of a lack of liquidity in the market may present difficulties in such a calibration over other time frames.

Inspection of the prices presented in table (1) illustrates the primary weaknesses of the model in relation to the dynamics of the NSW electricity price underlying. The model in its current form is unable to account for peak/off-peak or seasonally effects. This is evident in the SDE of the underlying process presented in section (3.1). The model has a constant mean reversion rate, mean reversion level and volatility. As such, the dynamics of the process represented by the SDE will make no distinction based on time of day or time of year. This is evident in the displayed model premia. Contracts that have the same strike price and time frame are valued with very similar premia for peak, off-peak or flat periods. This is contrary to the broker quotes and the general understanding of market participants.

The lack of seasonality in the model can be observed in contracts valuations such as the Q304 \$300 Flat Cap and the Q404 \$300 Flat Cap. The SDE has valued these contracts with almost the same premium, however, the empirical data reflect the fact that Q3 contains a lot of winter volatility while Q4 is a comparatively flat quarter.

Due to these factors, further work to adopt an SDE approach should extend the form presented in section (3.1) to allow such peak/off-peak and seasonality effects and in the representation of the underlying. One way to incorporate such effects would be through the incorporation of a time-dependent volatility term such presented below:

$$\frac{dS_t}{S_t} = \alpha [\mu - \ln(S_t)] dt + \sigma(t) dW_t$$

The function $\sigma(t)$ should be in a continuous functional form that allows some form of cyclic volatility function through the time of day and months of the year. A higher volatility during peak times of the day and months of the year would naturally provide the required differentiation in option prices and account for the higher prices observed in the underlying during these periods. A sinusoidal functional form with frequencies calibrated to seasonal effects and time of the day can be considered.

A further addition to provide a more realistic model of the underlying would be to include jump diffusion terms in the process to model extreme price events. While there was not a wide range of cap strike prices present in the broker data, the incorporation of jump diffusion would be expected to improve the robustness of valuing contracts over a wider range of strike prices.

The calibrated parameters for the single factor model are as follows:

$$\begin{array}{ccc} \alpha & \mu & \sigma \\ 8.25 & 4.58 & 6.43 \end{array} \quad (13)$$

These parameters can be understood when considering the effect of the lack of jump diffusion. The value of μ represents an electricity price mean reverting

level of \$97.51. While this is significantly greater than the mean reverting level of electricity spot price, the model has adopted this to compensate for the lack of a representation of extreme price occurrences in the model of the underlying. One potential strength of such an approach where parameters are calibrated to market data is that parameters will be determined such that reasonable interpolation of contract prices is obtained, even if the dynamics of the underlying are quite different from those of the SDE.

The parameters determined for the two-factor model are as follows:

$$\begin{array}{ccccc}
 \sigma & \alpha_\delta & \bar{\delta} & \sigma_\delta & \rho \\
 3.66 & 5.54 & 1.27 & 0.05 & -0.4
 \end{array}$$

Out of the three models, the two-factor model displayed the worst performance in calibrating to the broker data. This may imply a lack of significance of the convenience yield concept for this industry. It is well recognised that extreme price occurrences in the electricity market are typically due to extreme weather conditions causing excessive load. In such a situation, interconnects between regions may become constrained and as such, given regions are unable to import additional electricity from other regions in the NEM. The generation capacity of the base load generation plants is exceeded and the price moves into the excessive costs associated with peaking generators.

This situation has very little to do with the cost, or convenience of storing the fuels for generators but has much more to do with the infrastructure of generating plants in regions and interconnects as related to regional demands. The convenience yield concept adopted in the two-factor model is designed to model the effect of a yield generated by holding the fuels of generating plants. As such the model may not be a strong direction for further investigation into SDE approaches, but rather emphasis may be more relevant to the extensions relating to the single factor model discussed above.

4.3 General Discussion on the Fundamental Differences for the Two Approaches

The above results illustrate that both approaches adopted displayed comparable performance and each method may adopt extensions to improve its performance. In terms of the cashflow at risk Monte-Carlo methodology, improvements could be made by basing valuations on different calibrated risk premia for different contract properties (i.e. separate calibration for peak vs. off-peak contracts and different ranges of strike price). In the case of the SDE approach, further work would adopt models with properties that are more reflective of the underlying, such as time dependent volatility and jumps.

The distinction as to which approach is more appropriate is therefore dependent on the requirements of the valuation. One major distinction adopted in the two valuation approaches tested in this paper is whether the models are calibrated to contract premia or estimated from historical data. In these tests we primarily estimated the cash-flow at risk Monte-Carlo methodology from

historical data. Adopting such an estimation procedure should typically result in a model representation that is more reflective of the underlying process. The SDE approach was entirely calibrated to broker premia here and noting that the model of the underlying represented by the calibrated parameters would not be considered to model the properties of electricity spot price accurately. The approach of calibrating to market data does however allow a form of regression surface to be fitted to the available contract premia and facilitate a form of interpolation/extrapolation of quoted premia to other contracts of similar characteristics. As such, a process of market calibration would be expected, with further research, to achieve better pricing of contracts from available contract premia.

In the case of estimating models from the historical underlying data, it could be expected to establish a process that retains the structural properties of the underlying to a higher degree. As such, scenarios where pricing should indicate a potential exposure or cost incurred from holding a contract may be more relevant here.

Despite this distinction, forms of SDEs that are more indicative of the properties of the underlying, incorporating time of day, seasonality, and extreme events may prove to be more indicative of the properties of the historic underlying than observed here in the SDE approach here, even when calibrated. While it would be reasonable to argue that a model calibrated to market quotes would form better regressions of premia of similar contracts and models estimated from history would provide a stronger reflection of the structural properties of spot prices, a reasonable assessment of the quality of a model may be the degree to which these approaches converge. That is a market calibrated model should ultimately provide a reasonable reflection of historical spot prices and a historically estimated model should provide a reasonable method of pricing options.

Other distinctions between the two approaches adopted are that the cash-flow at risk approach is based on a Monte-Carlo method while the SDEs provided closed form pricing formulas. The primary implication here is that a Monte-Carlo approach requires extensive processing time to provide results while closed form solutions are potentially very fast for valuation. Additionally, the closed form solutions adopted allow the standard calculations of Greeks, while some thought would have to be put in to how to achieve such metrics with the Monte-Carlo approach.

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