

# Inflation modelling in the Australian Market

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**Determined** to be different

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**Determined** to be different

# Outline

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- What is inflation and inflation hedging?
- Inflation Bonds
- Inflation Swaps
- Building an Inflation Curve
- Convexity and Seasonality
- Pricing Inflation Derivatives



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# Outline

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# What is Inflation?

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- Inflation is the change in price of a basket of goods and services.
- The basket represents the costs of a typical consumer.
- It is represented by an index, based at 100 at some point in time.
- In Australia, the quarterly Consumer Price Index (CPI) was based at 100 in 1989-1990 (the average for the four figures that year is 100.0).
- In the UK, the monthly RPI and various LPIs (LPIs are collared between e.g. 1.0 and 1.05 times the previous figure).
- In the US there is CPI-U (non-seasonally adjusted CPI for urban consumers), and the core CPI ex food and energy.
- In the Eurozone there is monthly HICP (Harmonized Index of Consumer Prices) and HICPxT (ex Tobacco).



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# What is Inflation Hedging?

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- Inflation hedging is the transferring of inflation exposure to a preferred risk.
- **What are Inflation Swaps?**
- CPI swaps are interest rate transactions that allow counterparties to swap a floating CPI-linked obligation for a fixed or floating interest rate obligation.

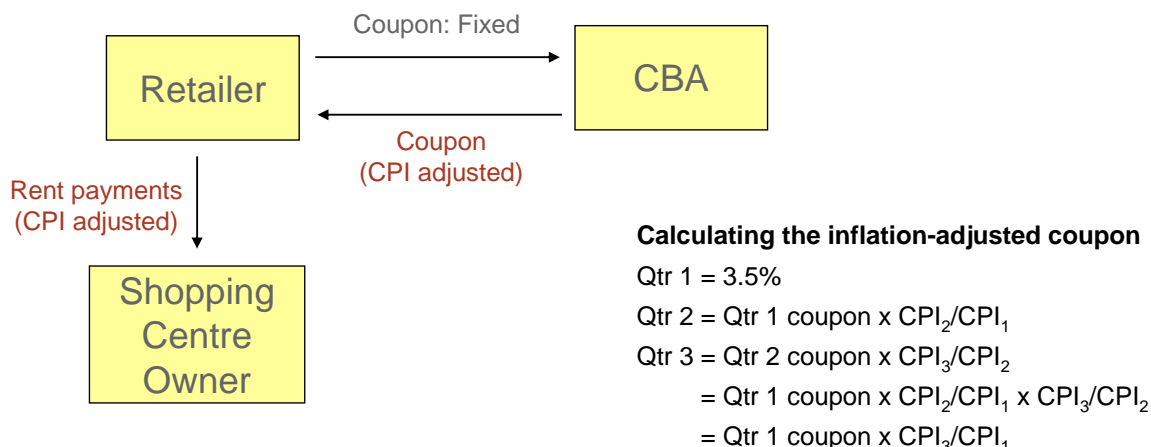


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# 1. Fixing an Inflation-linked Expense

Solution:

- The Retailer swaps an uncertain change in inflation for a fixed change in inflation.

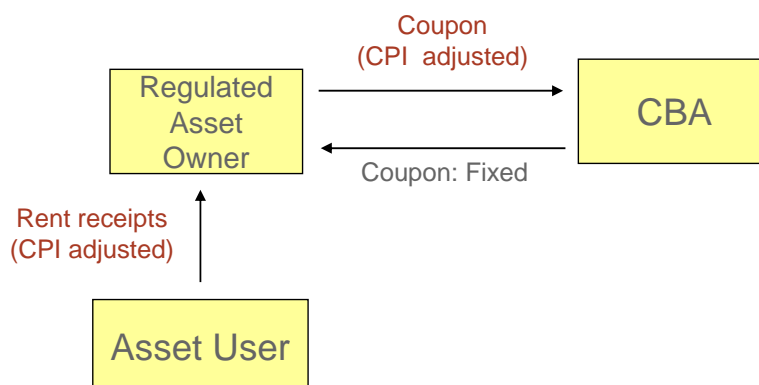


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# 2. Fixing Inflation-linked Revenue

Solution:

- Transact a fixed for floating CPI swap where the CPI-linked revenue is fixed.
- A known revenue flow makes it easier for the company to bid a higher price for the asset.



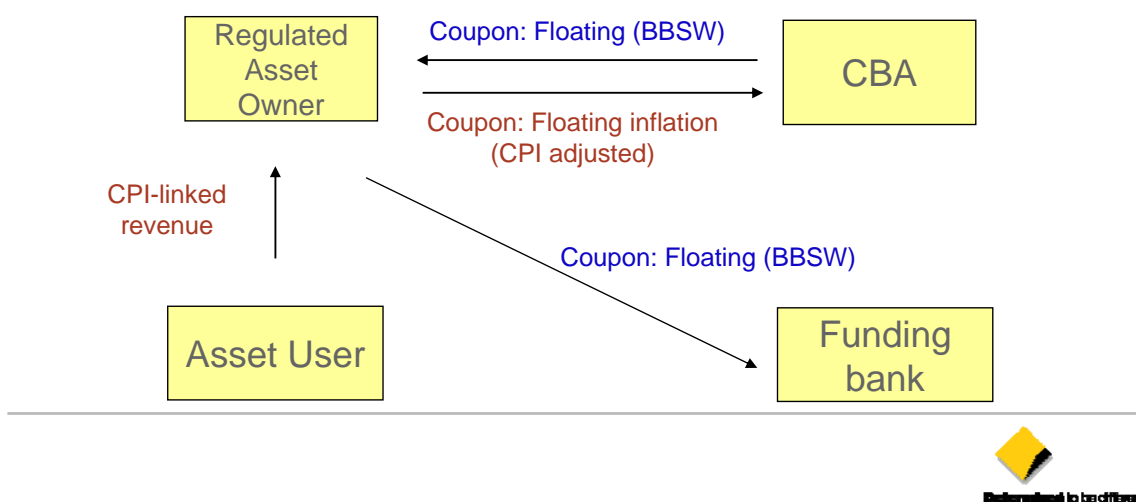
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### 3. Nominal Interest Expense Swapped for CPI

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Solution:

- Swapping BBSW payments for an inflation-linked exposure.
- Allows diversified financing arrangements to remain in place.
- Correlates interest liabilities with inflation-linked revenue.



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### Period on Period Inflation

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A second common representation is annualised Period on Period inflation:

- Year on Year:  $YoY_{Dec08} = CPI_{Dec08}/CPI_{Dec07} - 1$
- Qtr on Qtr:  $QoQ_{Dec08} = 4*(CPI_{Dec08}/CPI_{Sep08} - 1)$
- Month on Month:  $MoM_{Dec08} = 12*(CPI_{Dec08}/CPI_{Nov08} - 1)$

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# Consumer Price Index: AUD Historic Data

- Calculated by the ABS
- All Capital Cities Weighted Average
- Available back to 1850

Quarter	CPI
Mar-02	136.6
Jun-02	137.6
Sep-02	138.5
Dec-02	139.5
Mar-03	141.3
Jun-03	141.3
Sep-03	142.1
Dec-03	142.8
Mar-04	144.1
Jun-04	144.8
Sep-04	145.4
Dec-04	146.5
Mar-05	147.5
Jun-05	148.4
Sep-05	149.8
Dec-05	150.6
Mar-06	151.9
Jun-06	154.3
Sep-06	155.7
Dec-06	155.5
Mar-07	155.6
Jun-07	157.5
Sep-07	158.6
Dec-07	160.1
Mar-08	162.2
Jun-08	???

Source:  
ABS



## AUD Historic Data

- Spikes
- Deflation
- Mean reversion?
- Seasonality?

Quarter	Release		Annualised QoQ inflation rate	Annualised SoS inflation rate	YoY inflation rate
	Date	CPI			
Mar-00	26-Apr-00	125.2	3.55%	2.92%	2.79%
Jun-00	26-Jul-00	126.2	3.19%	3.38%	3.19%
Sep-00	25-Oct-00	130.9	14.90%	9.11%	6.08%
Dec-00	24-Jan-01	131.3	1.22%	8.08%	5.80%
Mar-01	25-Apr-01	132.7	4.27%	2.75%	5.99%
Jun-01	25-Jul-01	133.8	3.32%	3.81%	6.02%
Sep-01	24-Oct-01	134.2	1.20%	2.26%	2.52%
Dec-01	23-Jan-02	135.4	3.58%	2.39%	3.12%
Mar-02	24-Apr-02	136.6	3.55%	3.58%	2.94%
Jun-02	24-Jul-02	137.6	2.93%	3.25%	2.84%
Sep-02	23-Oct-02	138.5	2.62%	2.78%	3.20%
Dec-02	22-Jan-03	139.5	2.89%	2.76%	3.03%
Mar-03	23-Apr-03	141.3	5.16%	4.04%	3.44%
Jun-03	23-Jul-03	141.3	0.00%	2.58%	2.69%
Sep-03	22-Oct-03	142.1	2.26%	1.13%	2.60%
Dec-03	28-Jan-04	142.8	1.97%	2.12%	2.37%
Mar-04	28-Apr-04	144.1	3.64%	2.81%	1.98%
Jun-04	28-Jul-04	144.8	1.94%	2.80%	2.48%
Sep-04	27-Oct-04	145.4	1.66%	1.80%	2.32%
Dec-04	26-Jan-05	146.5	3.03%	2.35%	2.59%
Mar-05	27-Apr-05	147.5	2.73%	2.89%	2.36%
Jun-05	27-Jul-05	148.4	2.44%	2.59%	2.49%
Sep-05	26-Oct-05	149.8	3.77%	3.12%	3.03%
Dec-05	25-Jan-06	150.6	2.14%	2.96%	2.80%
Mar-06	26-Apr-06	151.9	3.45%	2.80%	2.98%
Jun-06	26-Jul-06	154.3	6.32%	4.91%	3.98%
Sep-06	25-Oct-06	155.7	3.63%	5.00%	3.94%
Dec-06	24-Jan-07	155.5	-0.51%	1.56%	3.25%
Mar-07	25-Apr-07	155.6	0.26%	-0.13%	2.44%
Jun-07	25-Jul-07	157.5	4.88%	2.57%	2.07%
Sep-07	24-Oct-07	158.6	2.79%	3.86%	1.86%
Dec-07	23-Jan-08	160.1	3.78%	3.30%	2.96%
Mar-08	23-Apr-08	162.2	5.25%	4.54%	4.24%
Jun-08	23-Jul-08	???	4.19%	4.75%	4.06%



# Inflation – The Economists View

Nominal yield (N) (increase in value) is made up of real yield (r), inflation ( $\pi$ ), inflation risk premium, liquidity premium, credit risk premium

$$1 + N = (1 + r)(1 + \pi)(1 + \rho)(1 + \lambda)(1 + \kappa)$$

Which is expanded, second order terms ignored and simplified to:

$$N = r + \pi$$

or

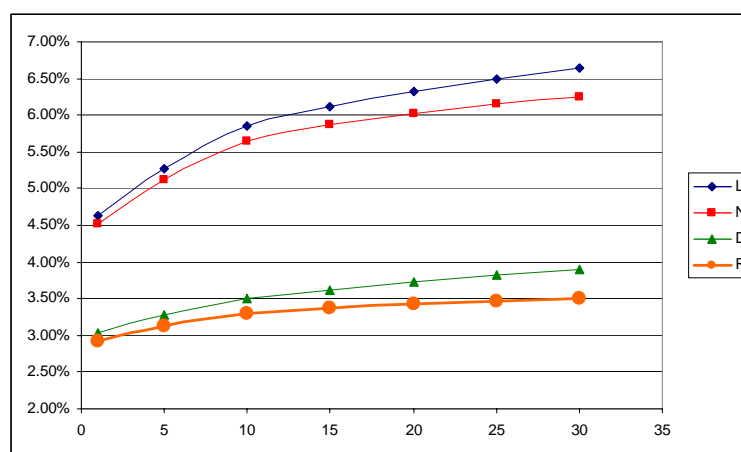
$$r = N - \pi$$

Real yield = Change in value – Change in purchasing power



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# Interest Rates Market Overview 1



**L** = LIBOR: Nominal Swap Rate

**N** = Nominal Yield, Govt Bond

**D** = Real Swap Rate

**R** = Real Yield, Govt Inflation Bond

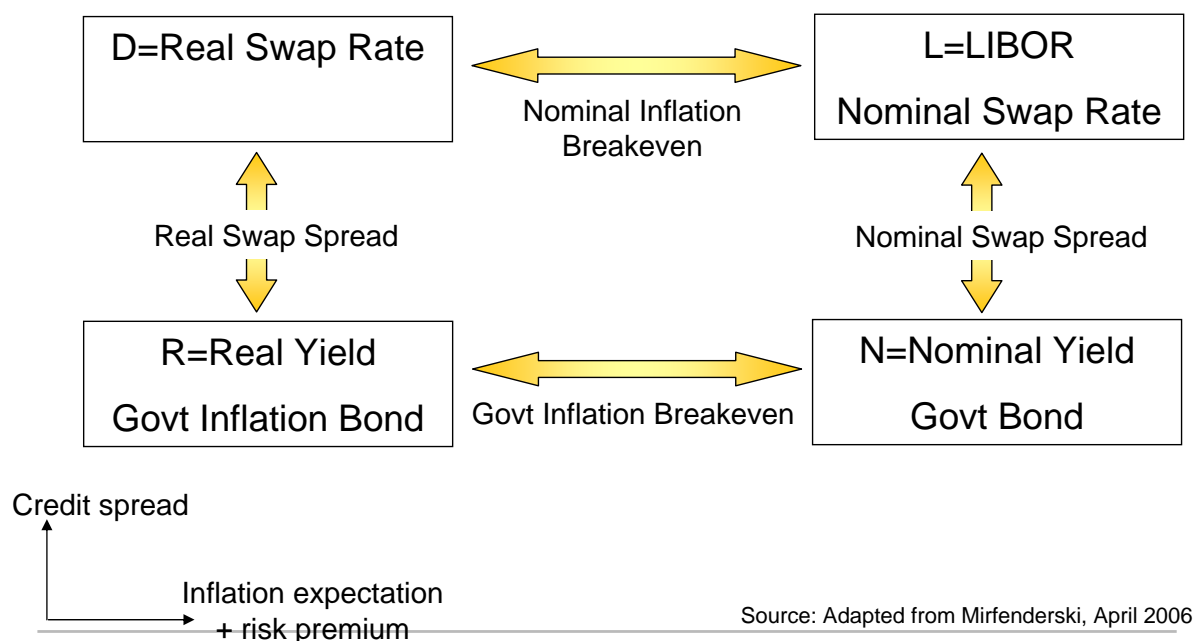
Source: Mirfenderski, April 2006



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# Interest Rates Market Overview 2

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## Outline

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# Government Inflation Linked Bonds

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Governments have issued inflation linked bonds (ILBs, linkers)

- Incentive to manage inflation (they pay inflation)
  - Reduce borrowing costs by the inflation risk premium
  - Support for Govt policy (in Australia in 1985 to complement the superannuation policy by offering an inflation proof asset to super funds)
  - Balance increased inflow from inflation based revenue
- 
- There are three Australian Commonwealth Government ILBs on issue, ILB2010, 2015, 2020.
  - ILB2005 Matured in August 2005, it was very illiquid as maturity approached.
  - Current Federal Govt policy is to issue no more ILBs.
  - QTC has issued a 2030 bond, and NSWTC a 2025 and 2035.
  - Corporate issues.



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## How do AUD ILBs work?

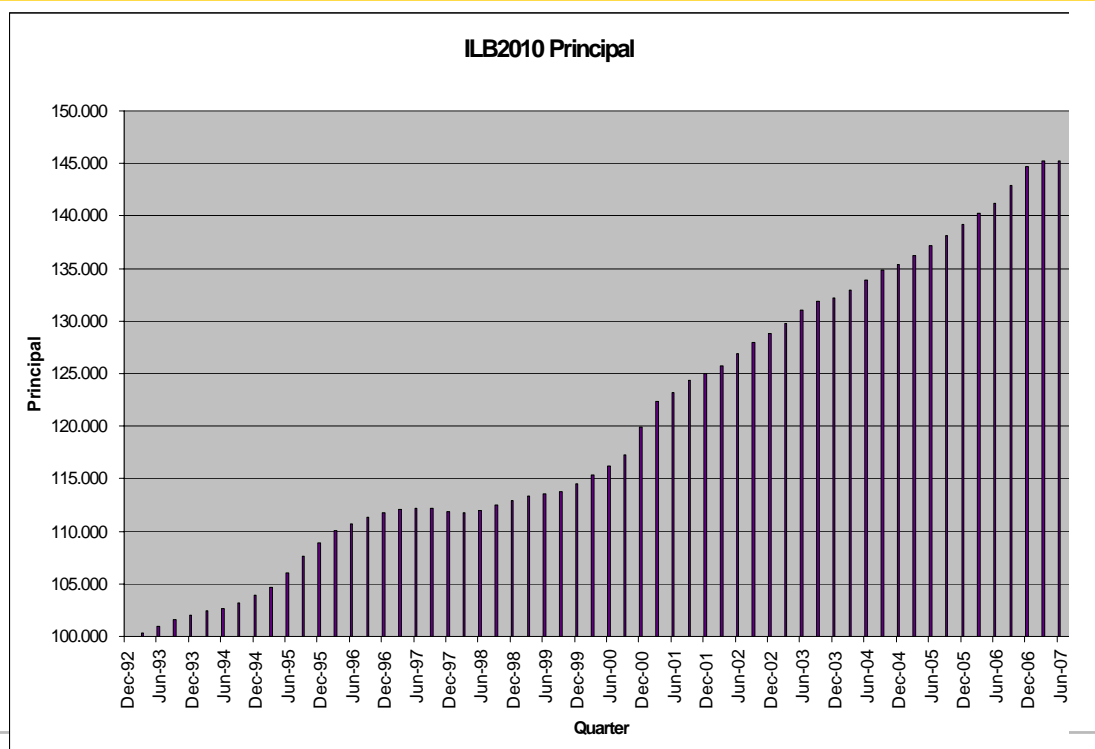
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- Pay a fixed coupon (4% per annum).
- Principal ( $K_t$ ) changes with (some measure of) the CPI.
- Principal can decrease if there is deflation.
- Base deflation adjusted (if during life  $K_t < 100.0$ , coupon is paid on 100.0 and excess deducted from future payments when  $K_t > 100.0$ ).
- Principal protected (return of principal floored at 100.0).
- If CPI figure is revised, bond principal and coupons are revised.



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# How do AUD ILBs work?



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## The Australian Calendar

- Measurement quarters end in Mar, Jun, Sep and Dec.
- The CPI figure is release in Apr, Jul, Oct, Jan (4th Wednesday of month).
- The coupon is paid on the new principal in May, Aug, Nov, Feb (20th day of the month).



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# Principal Calculation for AUD ILBs

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Next principal is previous principal increased by p

$$K_t = K_{t-1}(1 + p)$$

where

$$p = \frac{1}{2} \left( \frac{CPI_t}{CPI_{t-2}} - 1 \right)$$

p is the average % increase over two quarters. The averaging dampens spikes.

$CPI_t$  and  $CPI_{t-2}$  are the index values at two ends of a six month period.



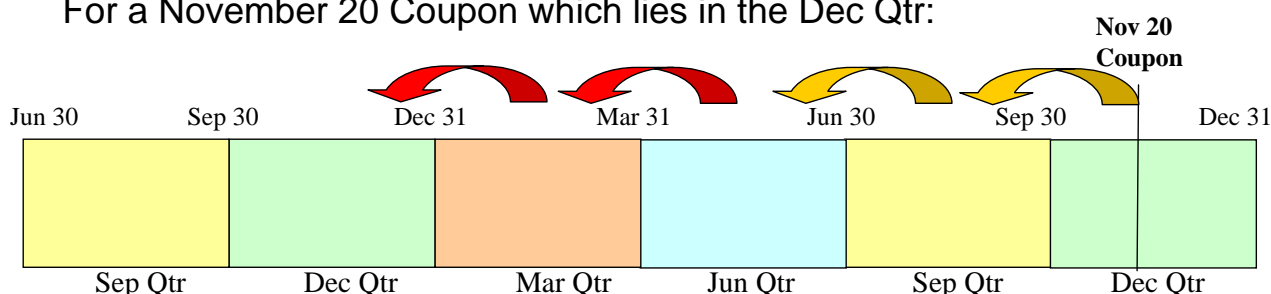
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## Example

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“the two quarters ending in the quarter which is two quarters prior to that in which the next interest payment falls”

For a November 20 Coupon which lies in the Dec Qtr:



Two quarters prior is the June quarter so  $CPI_t$  is the June CPI figure released in July and  $CPI_{t-2}$  is the figure released the previous January for the Dec Qtr the previous year.



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# Lags

The **lag** is the time difference between the latest released figure and the reference CPI being used.

- e.g. on Nov 20 the latest released figure is the September figure, we use the June figure so it is a 3 month lag.

Why lags?

If we have the calculation date for a derivative cashflow too soon after a release, and the release does not come out for unforeseen reasons, what would we do?

Getting the lags right is vital to inflation derivative pricing.



# Next principal is always known

Because of the lag the next cashflow is always known.

GRAB Corp CSHF

CUSIP:GG7150806 BOND PAYMENT SCHEDULE Page 1 / 1

AUSTRALIAN I/L ACGB 4 08/20/10 153.3420/153.3420 (2.98/2.98) AFMA

PRICE 153.342000 SETTLEMENT DATE 6/30/08 ISSUE 2/20/93 MATURITY 8/20/10

YIELD 2.9800 to M Maturity on 8/20/10 @ 100.000000

2)INFLATION ASSUMPTION 3.6009% FACE AMOUNT 1000.00 M

YIELD W/INFLATION ASSUMPTION 6.6300

Display C=Cashflow or P=Present Value @ % compounded 4/YR

\* -> Projected Cashflows

DATE	INTEREST	PRINCIPAL	DATE	INTEREST	PRINCIPAL
8/20/08	15047.00	0.00			
11/20/08 *	15181.00	0.00			
2/20/09 *	15316.00	0.00			
5/20/09 *	15452.00	0.00			
8/20/09 *	15589.00	0.00			
11/20/09 *	15727.00	0.00			
2/20/10 *	15867.00	0.00			
5/20/10 *	16008.00	0.00			
8/20/10 *	16150.00	1615000.00			

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000  
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2008 Bloomberg Finance L.P.  
6734-921-2 25-Jun-08 11:09:52

Source:  
Bloomberg

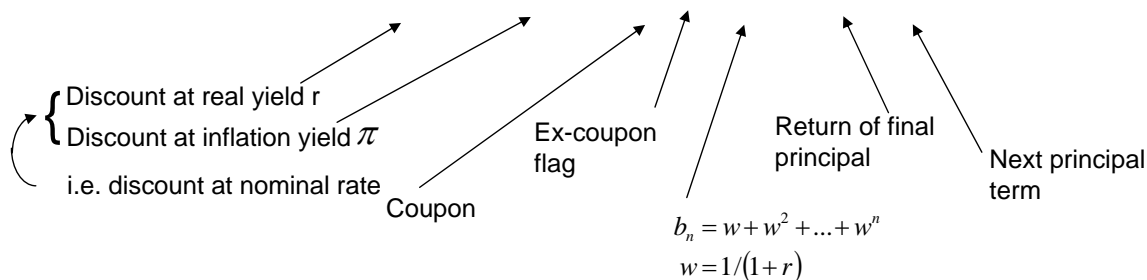


# RBA inflation bond formula

Nominal bond:  $P = v^{f/d} [g(x + a_n) + v^n]$        $a_n = v + v^2 + \dots + v^n, v = 1/(1+N)$

Inflation linked bond:

$$P = w^{f/d} (1 + \pi)^{-f/d} [g(x + b_n) + w^n] K_t$$



$$v = \frac{1}{1+N} = \frac{1}{(1+r)(1+\pi)} = w(1+\pi)$$



# Recent closes

## Last year:

ACGB 2010	6.27	Nominal	ACGB 2015	5.98	Nominal
ILB 2010	3.19	Real	ILB 2015	2.65	Real

Naïve implied inflation yield

3.08

Naïve implied inflation yield

3.33

## This week:

ACGB 2010	6.98	Nominal	ACGB 2015	6.61	Nominal
ILB 2010	2.99	Real	ILB 2015	2.87	Real

Naïve implied inflation yield

3.99

Naïve implied inflation yield

3.74



# Outline

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- What is inflation and inflation hedging?
- Inflation Bonds
- **Inflation Swaps**
- Building an Inflation Curve
- Convexity and Seasonality
- Pricing Inflation Derivatives



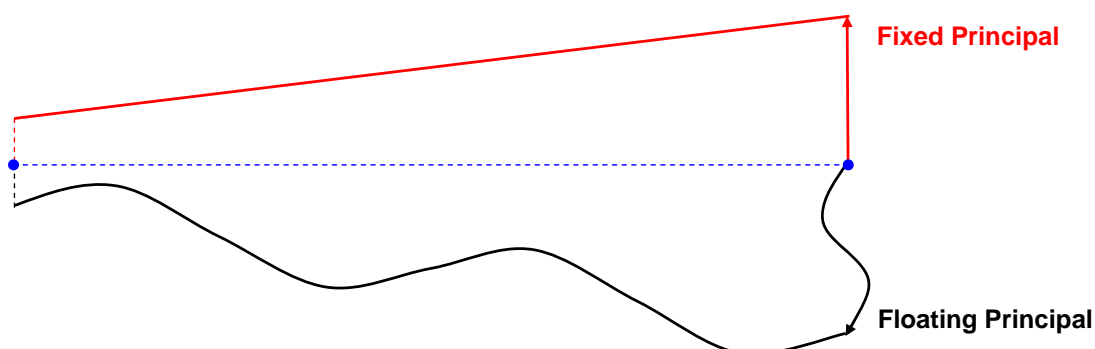
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## Standardised Inflation Swaps 1

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### 1. Zero Coupon Swaps

- Bullet exchange of indexed principal at maturity.
- The par rates for zero coupon inflation swaps are known as breakeven inflation (BEI).
- E.g. 2y 3.529%: Pay:  $P \cdot (1 + 3.529\%)^2$  Receive:  $P \cdot (\text{CPI}_2 / \text{CPI}_0)$

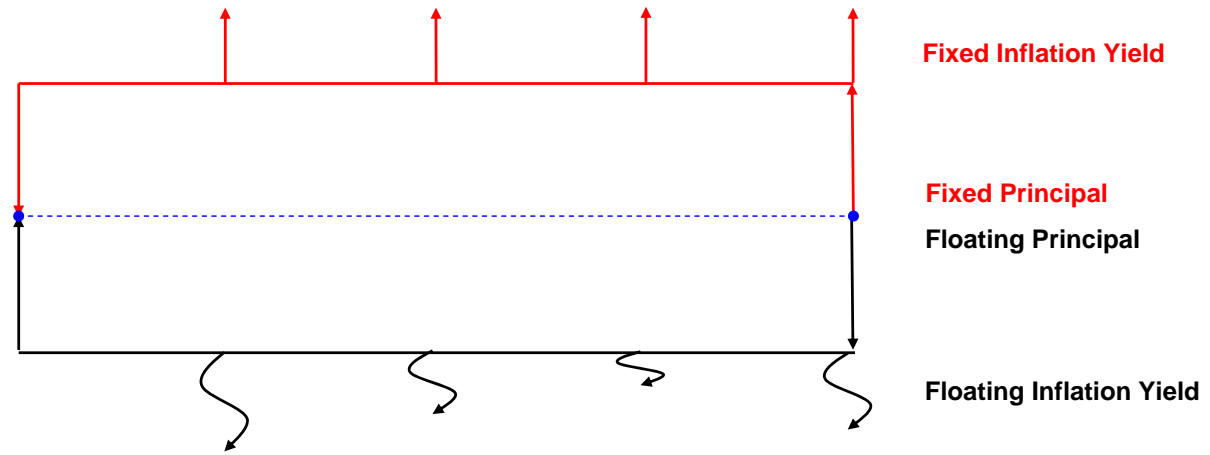


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# Standardised Inflation Swaps 2

## 2. Period on Period Swaps

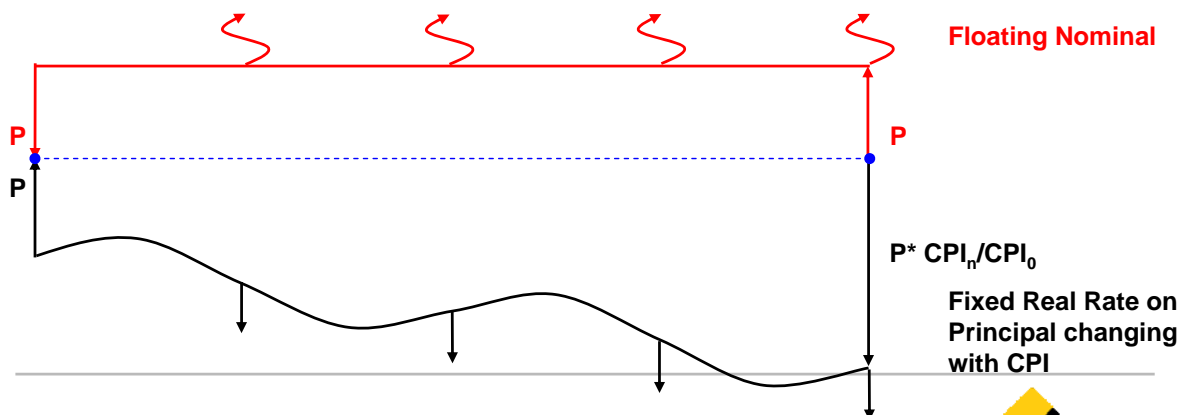
- Fixed versus floating annual coupon on fixed notional.
- Pay  $N \cdot (r_{fix}\%)$ , Receive  $N \cdot (CPI_n/CPI_{n-1} - 1)$



# Standardised Inflation Swap 3

## 3. Capital Indexed Swap

- Asset swap.
- Pay flows matching the ILB (Notional increases like the ILB).
- Receive BBSW + margin on a fixed notional;
- or receive a fixed rate on a fixed notional.

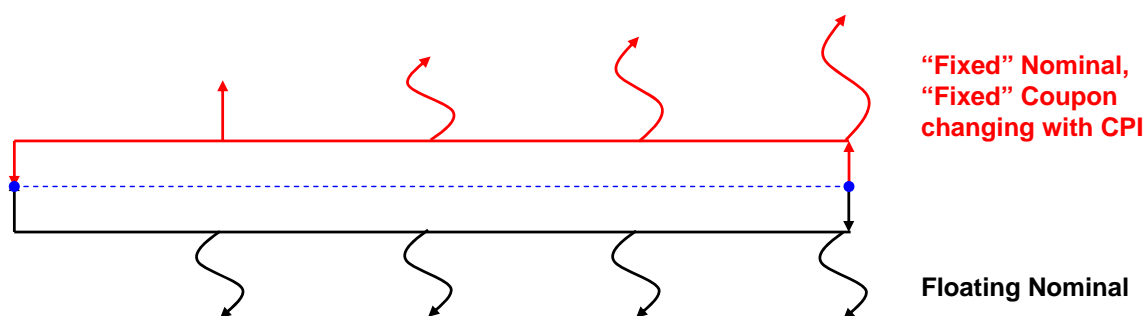


# Standardised Inflation Swap 4

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## 4. Indexed Annuity Swap

- Fixed notional.
- Pay a 'fixed' coupon which increases in line with inflation.
- Receive BBSW+ margin, or a flat fixed rate.



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# Building a forward AUD inflation curve 1

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- Not too different to building an IR yield curve.
- Like all yield curves
  - what set of discount factors and dates?
  - what interpolation method?
  - back out the prices of input market traded instruments
- A curve of CPI versus date.
- From this we can get expected forward CPI and forward inflation between two forward dates.
- We use forward inflation to construct swap cash flows.



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# Building a forward AUD inflation curve 2

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## ILBs as market inputs

**Method 1:** What constant increase in CPI per quarter backs out the market price of each bond?

CPIs from the first bond affect the principals to 2010 for the second.

Then we bootstrap to the end of 2015.



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# ILB Curve continued

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- Constant increases in CPI imply higher quarterly inflation in the short end of each bond.
- **Method 2:** What constant inflation rate in each quarter backs out CPIs which back out the prices of the bonds?
- **Method 3:** We looked at fitting shapes to the curve (e.g. Nelson Siegel style)
- All three methods give very similar results (except in the short end)



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# Semi-Government Bond Curve

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- Commonwealth Government ILB market extends to 12 years (2020).
- Semi-government CIBs at
  - 17y (2025 NSWTC)
  - 22y (2030 QTC)
  - 27y (2035 NSWTC)
- Should we use these to imply “risk-free” inflation expectations?
  - Liquidity
  - Semi-spread



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# Zero coupon swap curve

- Inputs are  $CPI_0$ , and the zero coupon breakeven inflation rates.

Then:

$$CPI_1 = CPI_0 (1 + \pi_1)$$

$$CPI_2 = CPI_0 (1 + \pi_2)^2$$

- Typical interpolation is linear interpolation on BEI or BEI\*time, so

$$\pi_{1.5} = (\pi_1 + \pi_2)/2 \quad \text{or} \quad \pi_{1.5} = \frac{(1 * \pi_1 + 2 * \pi_2)/1.5}{2}$$

$$\text{and} \quad CPI_{1.5} = CPI_0 (1 + \pi_{1.5})^{1.5}$$

- Base date is 15<sup>th</sup> of Mar, June, Sep, Dec.
- Base date rolls on CPI release dates.



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# AUD ZCIS curve

- Issued weekly by ICAP
- Average of contributor rates

Quote Browser: Item Address: QCPISWAPREF									
08:03 20JUN08 AUST INFLATION SWAP INTERBANK AVERAGE AU18992 CPISWAPREF									
SWAP VALUATION RATES AT 16:30 19/6/08 3YR 93.115 10YR 93.41									
AVERAGE:									
3YR	5YR	7YR	10YR	12YR	15YR	20YR	25YR	30YR	
3.83/93	3.87/97	3.90/00	4.00/10	4.06/16	4.09/19	4.13/23	4.15/25	4.17/27	
ELIGIBLE CONTRIBUTORS RATES									
3YR	5YR	7YR	10YR	12YR	15YR	20YR	25YR	30YR	
3.83/93	3.87/97	3.91/01	4.01/11	4.07/17	4.11/21	4.15/25	4.18/28	4.21/31	
3.84/94	3.90/00	3.90/00	4.00/10	4.08/18	4.11/21	4.15/25	4.15/25	4.18/28	
3.83/93	3.88/98	3.91/01	4.02/12	4.09/19	4.12/22	4.15/25	4.18/28	4.19/29	
3.87/97	3.90/00	3.94/04	4.00/10	4.04/14	4.10/20	4.15/25	4.17/27	4.18/28	
3.85/95	3.85/95	3.84/94	4.02/12	4.08/18	4.09/19	4.12/22	4.14/24	4.15/25	
3.82/92	3.87/97	3.93/03	4.00/10	4.03/13	4.07/17	4.11/21	4.14/24	4.15/25	
3.80/90	3.85/95	3.89/99	3.97/07	4.03/13	4.06/16	4.08/18	4.10/20	4.12/22	
3.82/92	3.87/97	3.89/99	4.00/10	4.06/16	4.09/19	4.12/22	4.15/25	4.16/26	
3.80/90	3.87/97	3.90/00	3.99/09	4.04/14	4.10/20	4.16/26	4.18/28	4.20/30	
3.87/97	3.85/95	3.89/99	3.99/09	4.05/15	4.09/19	4.13/23	4.15/25	4.17/27	
BANKS CONTRIBUTING TODAY: ABN ANZ CBA CITI DEU GS MBL NAB UBS JP									

Source:  
Reuters - ICAP



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# Why do we build a bond curve?

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- We trade interbank so don't we just need a ZCIS curve?
- ZCIS data comes out weekly, and is seen as broker rates.
- Bond data is daily, and comes from AFMA.
- In illiquid market, we build the daily bond curve and compare it to the ZCIS curve once a week (when data is available).
- ZCIS curve is becoming more liquid as the market evolves.



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# Pricing non-standard swaps

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- Estimate forward CPI from the curve, convert to forward inflation.
- Construct cash flows.
- Discount off the swap curve.
- An unrevised CPI figure is used under the standard ISDA (except in the case of manifest error).
- Note: Curves built from traded assets. Numeraire is the capitalisation factor. CPIs and inflation rates are ratios of these instruments and thus martingales. No convexity adjustment needed.



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# Convexity

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- Forward inflation is not a martingale under the risk free measure used to price inflation trades.

$$E\left\{\frac{CPI_t}{CPI_{t-1}}\right\} \neq \frac{E\{CPI_t\}}{E\{CPI_{t-1}\}}$$

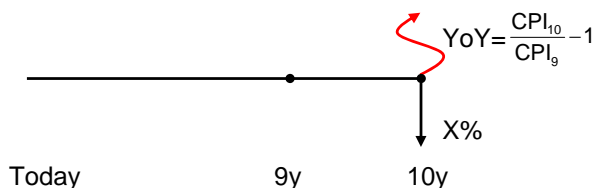
- This will affect forward inflation rates both in PoP swaps and as inputs into closed form option pricing.



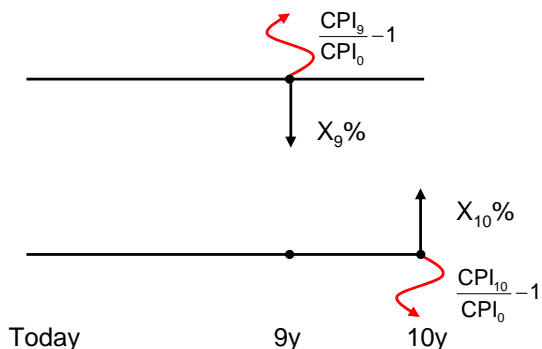
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# Convexity – intuitive explanation

- Say I enter an inflation FRA to pay floating YoY inflation on year 9 to year 10 (and receive fixed).



- A hedge (N.B. not a replication) for this is to sell a 9y ZCIS (pay 9y inflation) and buy a 10y ZCIS (receive 10y inflation).



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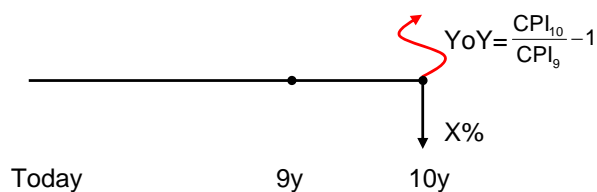
# Convexity

- The payoff for the 9y ZCS can be reinvested.
- If realised 9y came in low then I have money in hand. The argument goes that IF BEI is positively correlated to the nominal rate, then I reinvest at a lower rate. Conversely if 9y came in high, then I have to borrow at a higher rate to fund the payout for a year.
- Hence I expect to lose money and thus will pay less for the YOY: i.e. the expected forward rate is reduced (-ve adjustment).

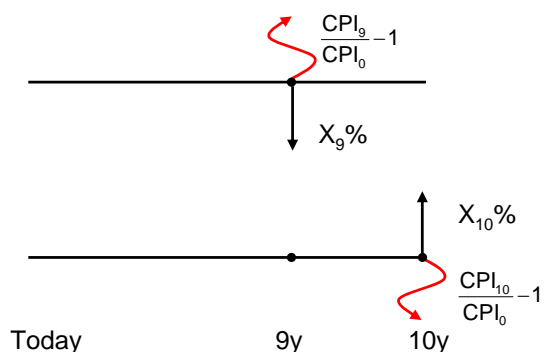


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# London



Forward Maturity	YoY adjustment
1	-0.6
5	-1.1
10	-2.4
15	-3.1
20	-3.6
25	-4.4
30	-5.2



Source: Benaben and Tabardel



# Convexity

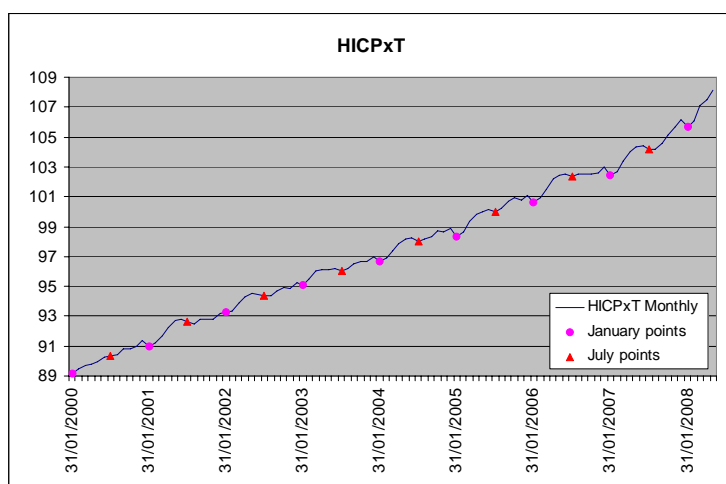
- Adjustment is model dependant.
- The standard measure used is the nominal forward measure, expectation of real ZCB prices then relies on the correlation of real and nominal (and vols, levels of each etc).
- Jarrow-Yildirim and Mercurio give closed form formulae.



# Seasonality

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- Seasonality is widely accepted to exist in the EUR, GBP and USD inflation markets.
- The RBA says there is no seasonality in AUD inflation.



Source: Bloomberg



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# Outline

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- What is inflation and inflation hedging?
- Inflation Bonds
- Inflation Swaps
- Building an Inflation Curve
- Convexity and Seasonality
- **Pricing Inflation Derivatives**



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# Pricing options

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- Options appear as deflation floors in notes or bonds, but one can just as well sell a pure cap or floor on Period on Period inflation.
- Simple model
  - Black style, assume inflation rate follows a normal distribution.
  - Volatility requires analysis (simplest rule of thumb takes vol of inflation as scaling factor x vol of nominal rate).
- Swaptions and bond options are sold in the major markets (OTC at this stage, standardisation starting to appear)



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## Approaches to pricing Inflation Options

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- Normal Black model (Bachelier formula) for caplets and floorlets.
- Jarrow and Yildirim (2003): three factor HJM model using a foreign currency analogy.
- Mercurio (2005): market model approach.
- Mercurio and Moreni (2006): market model with stochastic volatility.
- Kruse (2007): market model with lognormal CPI preserving the Fisher equation.



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# Jarrow-Yildirim: FX view of Inflation

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- CPI = exchange rate (between the real and nominal economies)
- Nominal yield  $N$  = domestic rate
- Real yield  $R$  = foreign rate
- In the continuously compounded world  $CPI_t = CPI_0 \exp((N-R)t)$
- Inflation =  $N-R$  is the cost of carry
  
- Usually the nominal and real rates are modelled as 1-factor HJM diffusion processes. CPI is modelled as a lognormal process.
- Enables us to write options on CPI and use closed form formulae.



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## Drawbacks with Jarrow-Yildirim

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- Smile effect not taken into account.
- Model relies on non-observable data (i.e. instantaneous rates).
- Difficulty of estimating historically the real rate parameters, especially volatility.



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# Market Models

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- Various market model approaches have been proposed by Mercurio (and others).
- Extended to incorporate stochastic volatility.
- Advantages:
  - More accurate calibration to market data.
  - Do not need to estimate volatility of real rates.
  - Includes stochastic volatility.
- Disadvantages:
  - Macroeconomic concepts of Fisher equation not reflected in the lognormal forward CPI processes.
  - Approximations less accurate for long maturities.



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# Alternative Models

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- Kruse (2007): Market model with stochastic volatility that preserves the Fisher equation.
- Macroeconomic models.



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# Outline

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- What is inflation and inflation hedging?
- Inflation Bonds
- Inflation Swaps
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- Convexity and Seasonality
- Pricing Inflation Derivatives



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# Inflation modelling in the Australian Market

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**Determined** to be different

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