Q-Group

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Designing "Global Macro" Investment Strategies with Asymmetry Preferences

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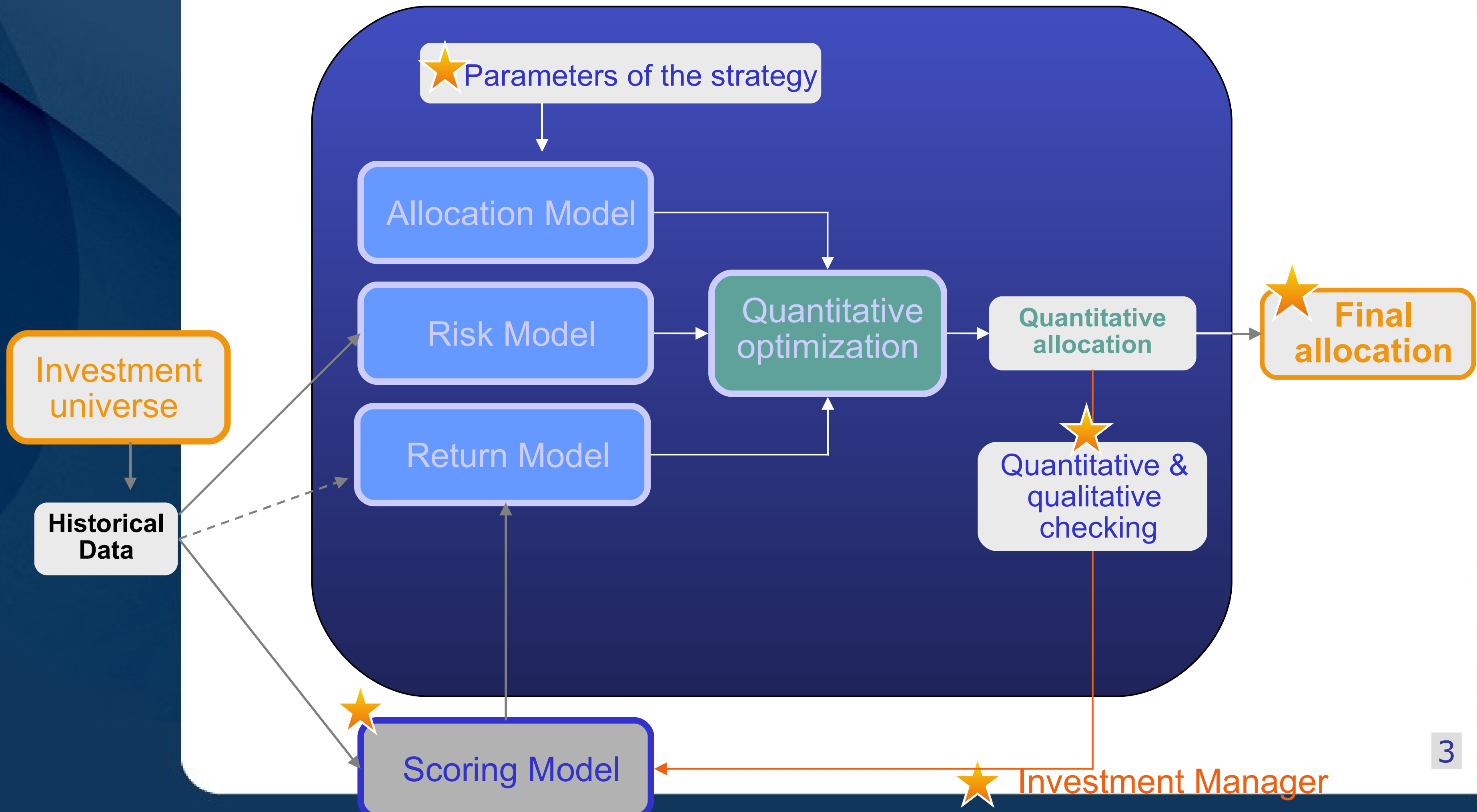
Outline

- **Modeling the strategy**
  - Global overview
  - Second order risk model (filtering)
  - Third order risk model
- **Designing the strategy in 3 steps**
  - Starting from “plain Markowitz”
  - Adding robust risk model
  - Adding asymmetry preferences
  - Compared analysis
- **A real life production : Quetzal by Osprey AM**

Modeling the investment strategy



Modelling risk: an optimization-driven filtering

Why do we need to filter matrices?

What do we mean by filtering?

- Reliable information/Noise
- Optimization point of view: controlling the condition number (or smallest eigenvalue)

How do we perform a conservative but reactive filtering?

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Modelling risk: an optimization-driven filtering

Why?

Historical covariance matrix is hardly a good estimate:

➔ Filtering noise / focus on the smallest eigenvalues

Statistical analysis shows the structural deficit of the empirical covariance matrix estimator to capture properly:

- the smallest contributions to risk
- i.e the smallest eigenvalues of the covariance matrix
- Or variances of main factors of the PCA

➔ highly dangerous for the design of optimal control decisions (optimization or simulation)

➔ typical situation when using noisy data

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Modelling risk: an optimization-driven filtering

Why?

Random matrix theory

The empirical correlation matrix $\mathbf{C}$ is constructed from the time series of price changes[a] $\delta x_i(t)$ (where i labels the asset and t the time) through the equation:

$$\mathbf{C}_{ij} = \frac{1}{T}\sum_{t=1}^{T} \delta x_i(t)\delta x_j(t). \qquad (0.1)$$

We can symbolically write Eq. (0.1) as $\mathbf{C} = 1/T\ \mathbf{M}\ \mathbf{M}^T$, where $\mathbf{M}$ is a $N \times T$ rectangular matrix, and ^{T} denotes matrix transposition. The null hypothesis of independent assets, which we consider now, translates itself in the assumption that the coefficients $M_{it} = \delta x_i(t)$ are independent, identically distributed, random variables [b]; the so-called random Wishart matrices or Laguerre ensemble of the Random Matrix theory[8,10]. We will note $\rho_C(\lambda)$ the density of eigenvalues of $\mathbf{C}$,

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Why?

Random matrix theory

L. Laloux, P. Cizeau, J-P. Bouchaud and M. Potters, *Noise Dressing of Financial Correlation Matrices*. Phys. Rev. Lett. 83, 1467 - 1470 , 1999.

Random Matrix theory[8,10]. We will note $\rho_C(\lambda)$ the density of eigenvalues of $\mathbf{C}$, defined as:

Density of eigenvalues for random matrices = full noise matrices

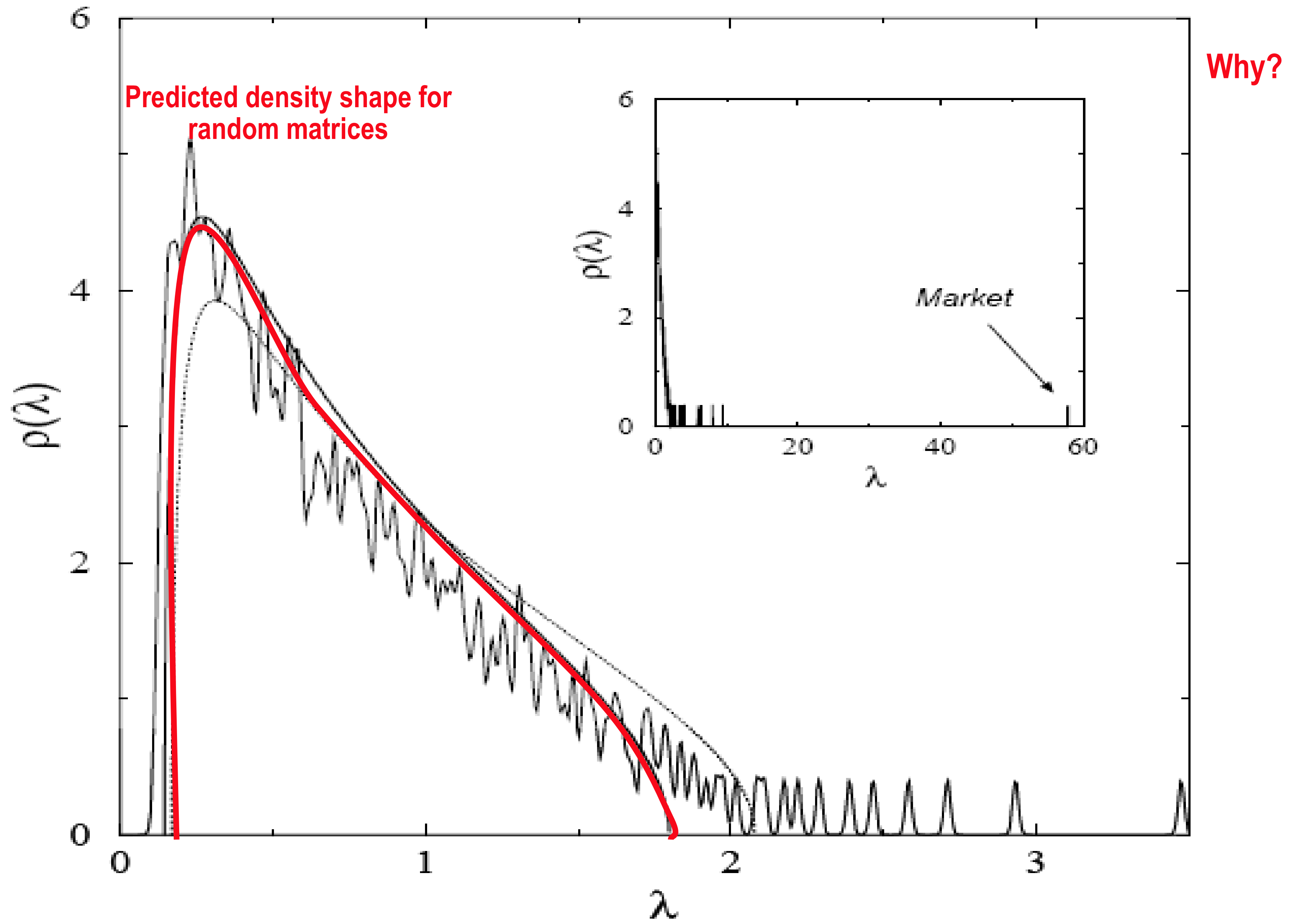
$$\rho_C(\lambda) = \frac{1}{N}\frac{dn(\lambda)}{d\lambda}, \qquad (0.2)$$

where $n(\lambda)$ is the number of eigenvalues of $\mathbf{C}$ less than λ.

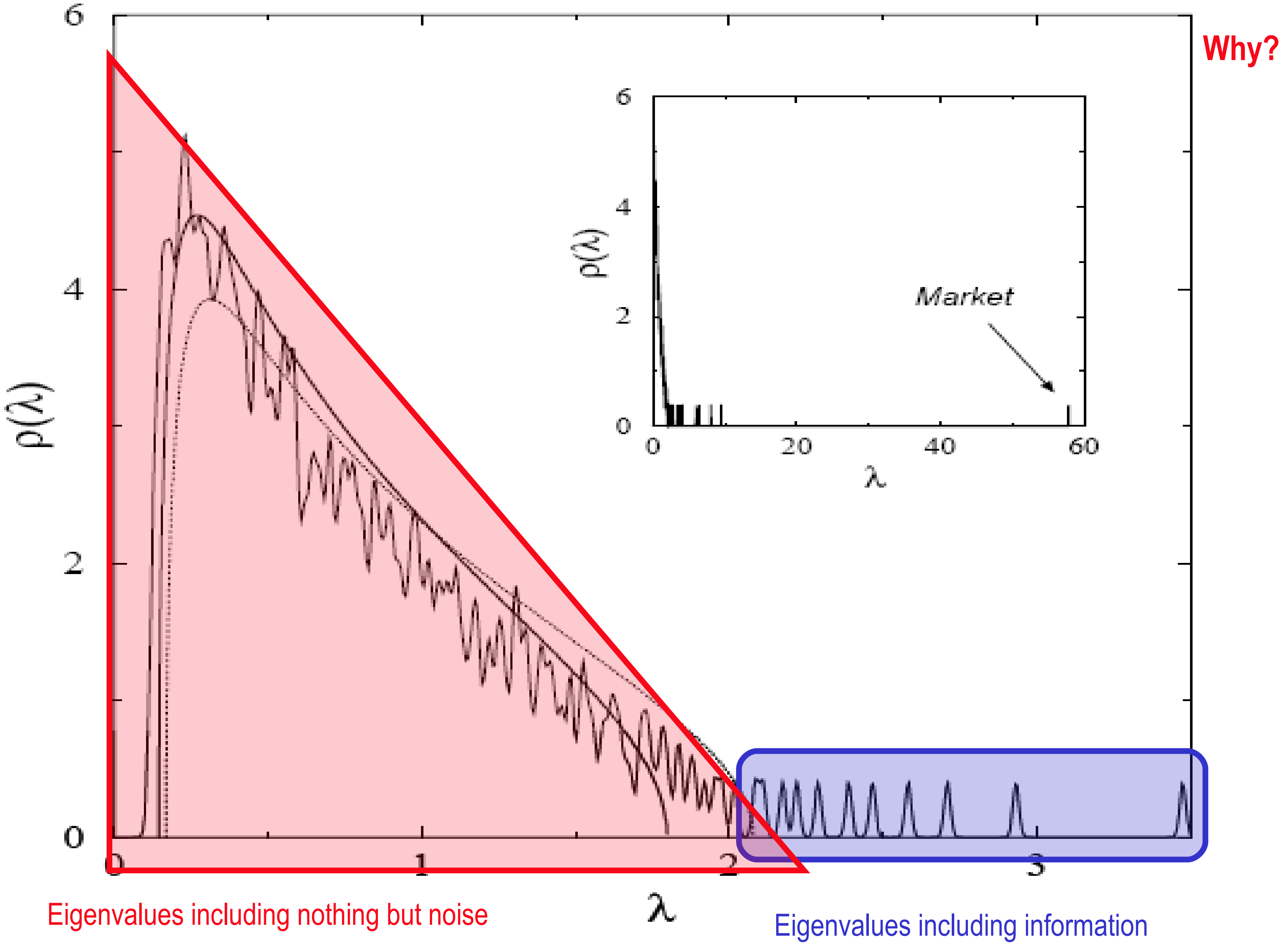
Interestingly, if $\mathbf{M}$ is a $T \times N$ random matrix, $\rho_C(\lambda)$ is self-averaging and exactly known in the limit $N \to \infty$, $T \to \infty$ and $Q = T/N \geq 1$ fixed [8,9], and reads:

$$\begin{aligned} \rho_C(\lambda) &= \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max}-\lambda)(\lambda-\lambda_{min})}}{\lambda}, \\ \lambda_{min}^{max} &= \sigma^2(1+1/Q \pm 2\sqrt{1/Q}), \end{aligned} \qquad (0.3)$$

with $\lambda \in [\lambda_{min}, \lambda_{max}]$, and where σ^2 is equal to the variance of the elements of $\mathbf{M}$ [9], equal to 1 with our normalisation. In the limit $Q = 1$ the normalised eigenvalue



Predicted density shape for random matrices
Why?
Market
ρ(λ)
λ



Why?
Market
Eigenvalues including nothing but noise
Eigenvalues including information

Modelling risk: an optimization-driven filtering

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The quality of the estimation of the smallest eigenvalue is bad when T is small

$$\rho_C(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max}-\lambda)(\lambda-\lambda_{min})}}{\lambda},$$
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Modelling risk: an optimization-driven filtering

Why?

Why underestimating small risk contributions is dangerous?

- Creating the illusion of portfolios with hudge ex ante sharpe ratios (factors linked to smallest eigenvalues)
- Then optimization will be driven by these noisy portfolios generating large turnovers and operational risks
- In fact, sensitivity analysis predicts this large turnovers (proportional to $\frac{1}{\lambda_{\min}}$)

$$\min_{\omega} \omega^t \Gamma \omega$$

$$\begin{cases} \rho^t \omega \geq \ell \\ \sum_i \omega_i = 1 \end{cases}$$

A small perturbation of ρ or Γ is magnified at the portfolio/decision level by a factor proportional to $\frac{1}{\lambda_{\min}}$.

L. El Ghaoui, F. Oustry and H. Lebret.
Robust Solutions to Uncertain Semidefinite Programs SIAM J. Optimization, volume 9, no. 1, 1998.

Modelling risk: an optimization-driven filtering

How?

Preserving "conservative" requirements while producing "reactive" matrices

- We want to capture market changes
- We want this matrix to produce robust portfolio decisions/deviations

➔ Reducing the estimation windows, incorporating implied information

➔ Filtering the noise due to the lack of data and improving the condition number

Like if you were designing a robust control law for an fighter aircraft, you need to react quickly but conservatively using very short term information through a very cautious filtering scheme

Modelling risk: an optimization-driven filtering

How?

A SDP formulation of the filtering problem:

$$\begin{cases} \min_{\Gamma \in \Omega} \|X - \Gamma\|_F^2 \\ \textit{subject to} \\ \quad \langle A_j, X \rangle_F = b_j,\ j = 1,\dots,m \\ \quad X \succeq \sigma^2 I \end{cases}$$

Level for the smallest eigenvalue

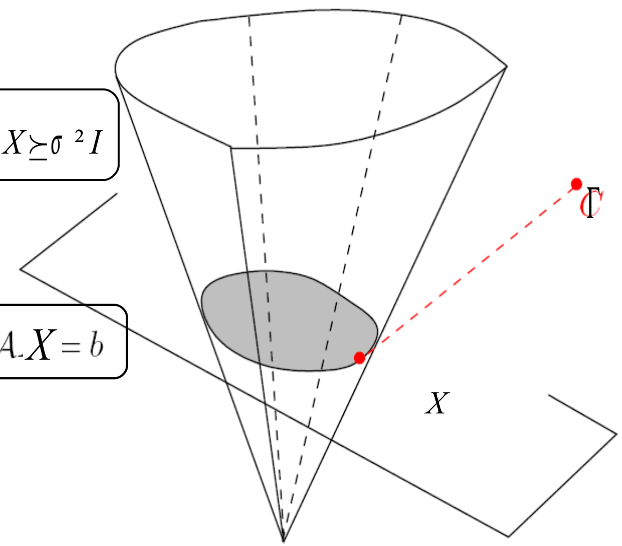
Filtering=

Enforcing low condition number

$X \succeq \sigma^2 I$

Ex: volatilities preservation:

$A.X = b$



$\Gamma_{ii} - \delta_i \leq X_{ii} \leq \Gamma_{ii} + \delta_i$ ← Possible evolution to $A(X) = [X_{11};\dots;X_{nn}]$

Modelling risk: an optimization-driven filtering

How?

A SDP formulation of the filtering problem:

The original risk matrix, target

$$\begin{cases} \min\limits_{\Gamma \in \Omega} \| X - \Gamma \|_F^2 \\ \textit{subject to} \\ \quad \langle A_j, X \rangle_F = b_j,\ j = 1,\ldots,m \\ \quad X \succeq \sigma^2 I \end{cases}$$

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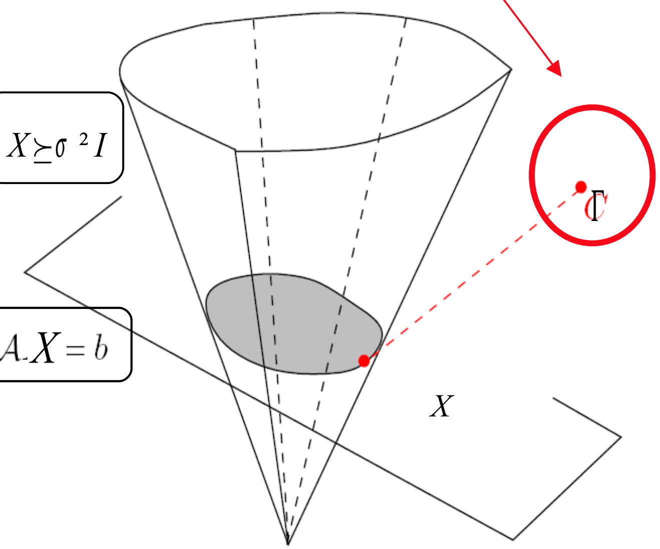

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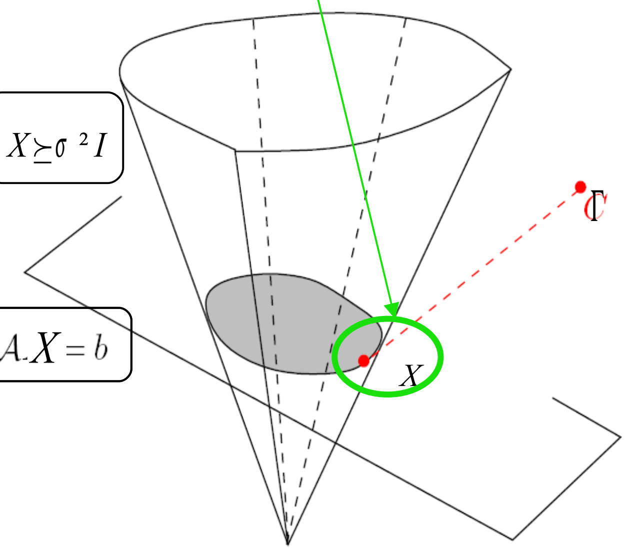
The new estimate

Level for the smallest eigenvalue

Filtering= Enforcing low condition number

$X \succeq \sigma^2 I$

Ex: volatilities preservation: $A.X = b$



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Minimizing the distance = projection

Level for the smallest eigenvalue

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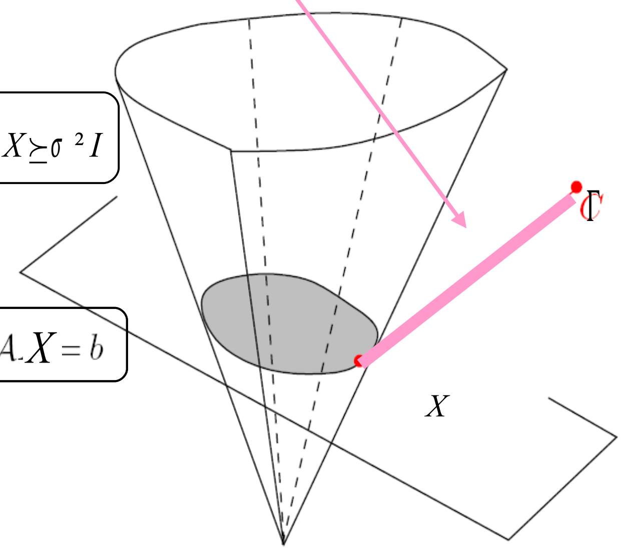

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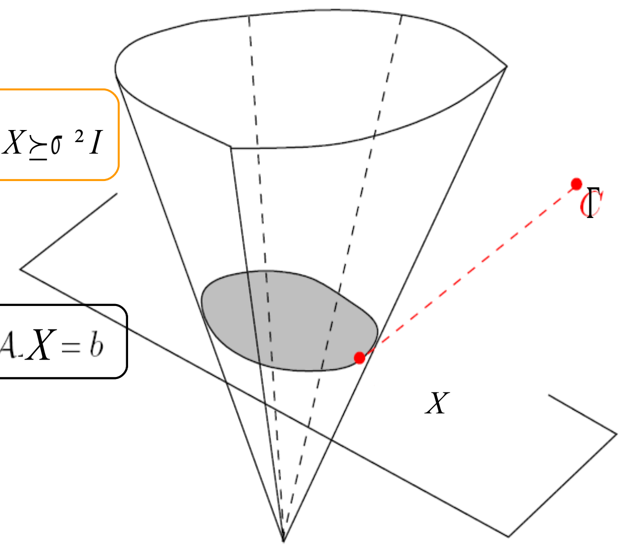

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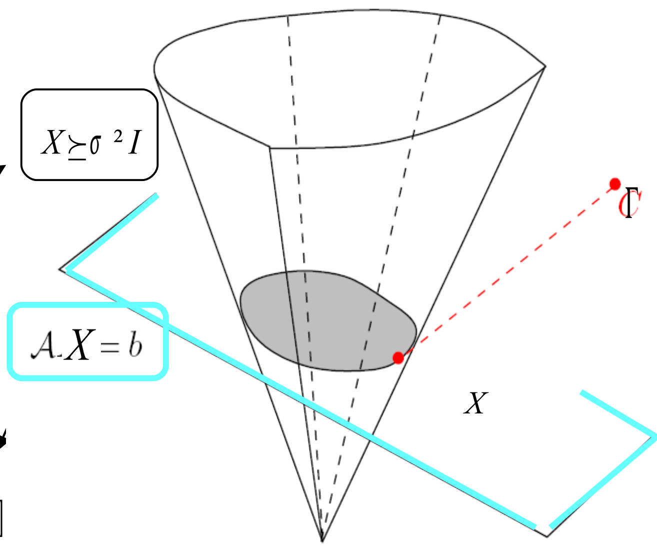

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Modelling risk: an optimization-driven filtering

How?

A formulation suited for various kind and constraints of real-life management:

- Multi asset-class strategy (risk aggregation)
- Asynchronous data
- Stress-testing matrices (risk management)
- Prior views or intervals on part of the matrix (volatilities, local markets matrix constraints, correlations, implied information)

📖 Pape Momar Ndiaye, François Oustry, Véronique Piolle. *Semidefinite optimisation for global risk modeling*, Journal of Asset Management 7, 142-153, 2006".

Modelling allocation: add asymmetry preference

When taking a robust control angle, you do not use statistics to produce a fine description of your system; you rather use statistics to shape your decision process, here the shape of your risk metric as a function of your decision variables : we want to include asymmetry preference while preserving convexity!!

Modelling allocation: add asymmetry preference

A convenient scalar product on our constraint-space $\mathcal{S}_n$ is the ordinary dot-product in $\mathbb{R}^{n^2}$:

$$(\mathcal{S}_n \times \mathcal{S}_n) \ni (X, U) \mapsto \langle X, U \rangle := \operatorname{tr} XU = \sum_{i,j=1}^{n} X_{ij} U_{ij} \,,$$

also called the trace-product. The reason it is so convenient is the following relation: for all $X \in \mathcal{S}_n$ and $u \in \mathbb{R}^n$,

$$\left[\textstyle\sum_{i,j=1}^{n} X_{ij} u_i u_j =\right] u^\top X u = \langle X, uu^\top \rangle \,, \qquad (11)$$

which can be checked in a straightforward way (note: $(uu^\top)_{ij} = u_i u_j$); it will be used continually.

Modelling allocation: add asymmetry preference

📖 E. Jondeau and M. Rockinger, *Optimal Portfolio Allocation Under Higher Moments*, Working Paper N. 108, December 2002 (Revised: January 2004). Available at http://www.banque-france.fr/gb/publications/ner/1-108.htm

📖 C.R. Harvey, M. Liechty, J. Liechty and P. Müller. *Portfolio Selection with Higher Moments.* (December 13, 2004). Available at SSRN: http://ssrn.com/abstract=634141

$$\begin{cases} \min_{\omega} \ \omega^T \Gamma \omega \\ s.t. \\ \qquad \sum_{i,j,k} \omega_i \omega_j \omega_k H_{ijk} \geq h \\ \qquad \rho^T \omega \geq \ell \\ \qquad \omega \in \Delta \end{cases} \qquad \begin{cases} \min_{\omega} \ \omega^T \Gamma \omega \\ s.t. \\ \qquad \left\langle \sum_k \omega_k H_{[k]}, W \right\rangle \geq h \\ \qquad \rho^T \omega \geq \ell \\ \qquad \omega \in \Delta \\ \qquad W = \omega \omega^T \end{cases}$$

$$where \ H_{[k]} \ is \ the \ N \times N \ matrix \ whose \ i,j \ element \ is \ H_{ijk}$$

Modelling allocation: add asymmetry preference

Optimization with third order moment constraint

$$\begin{cases} \min\limits_{\omega} \ \langle \Gamma, W \rangle \\ s.t. \\ \qquad \langle H, Z \rangle \geq h \\ \qquad Z \succeq (W, \omega) \otimes (W, \omega)^{*} \\ \qquad \rho^{T} \omega \geq \ell \\ \qquad \omega \in \Delta \\ \qquad W \succeq \omega \omega^{T} \end{cases}$$

$$where \ \ Z_{ijk} \ \ is \ the \ N \times N \times N \ third \ \ order \ \ portolio \ \ tensor$$

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Diversified (GM) Investment Universe

From “Plain Markowitz” to “Exotic Convex Optimization”

- **Plain Markowitz**
  - Performance model : « trend following »
  - Risk model : empirical covariance matrix
  - Allocation model : Minimize Volatility s.t. Perf constraint
- **Filtered Markowitz**
  - Performance model : « trend following »
  - Risk model : filtered covariance matrix
  - Allocation model : Minimize Volatility s.t. Perf constraint
- **Exotic Convex Optimization**
  - Performance model : « trend following »
  - Risk Model : Filtered covariance matrix + Higher moments/CVaR …
  - Allocation model : Minimize Volatility s.t. Perf constraint & Asymmetry preferences (third order moment, CVaR constraints, …)

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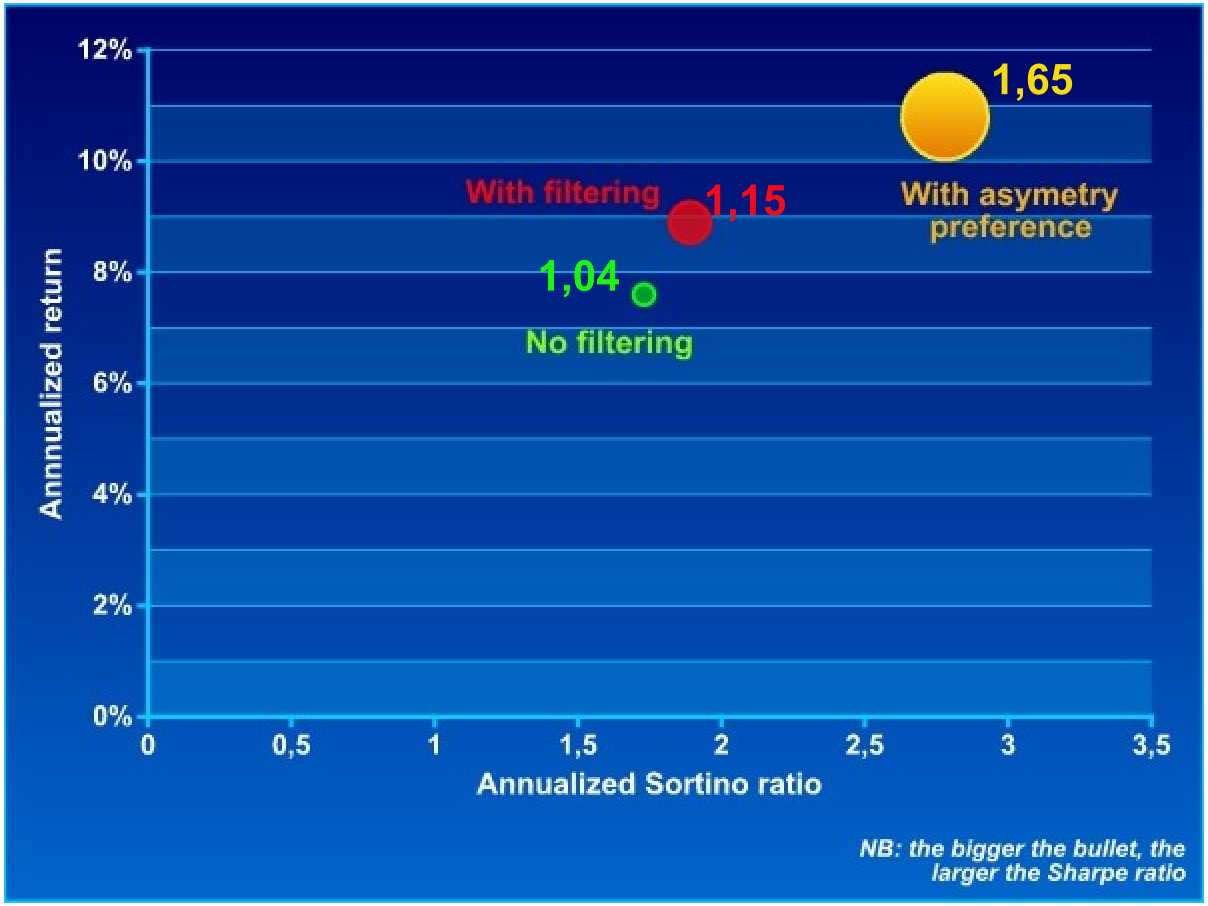
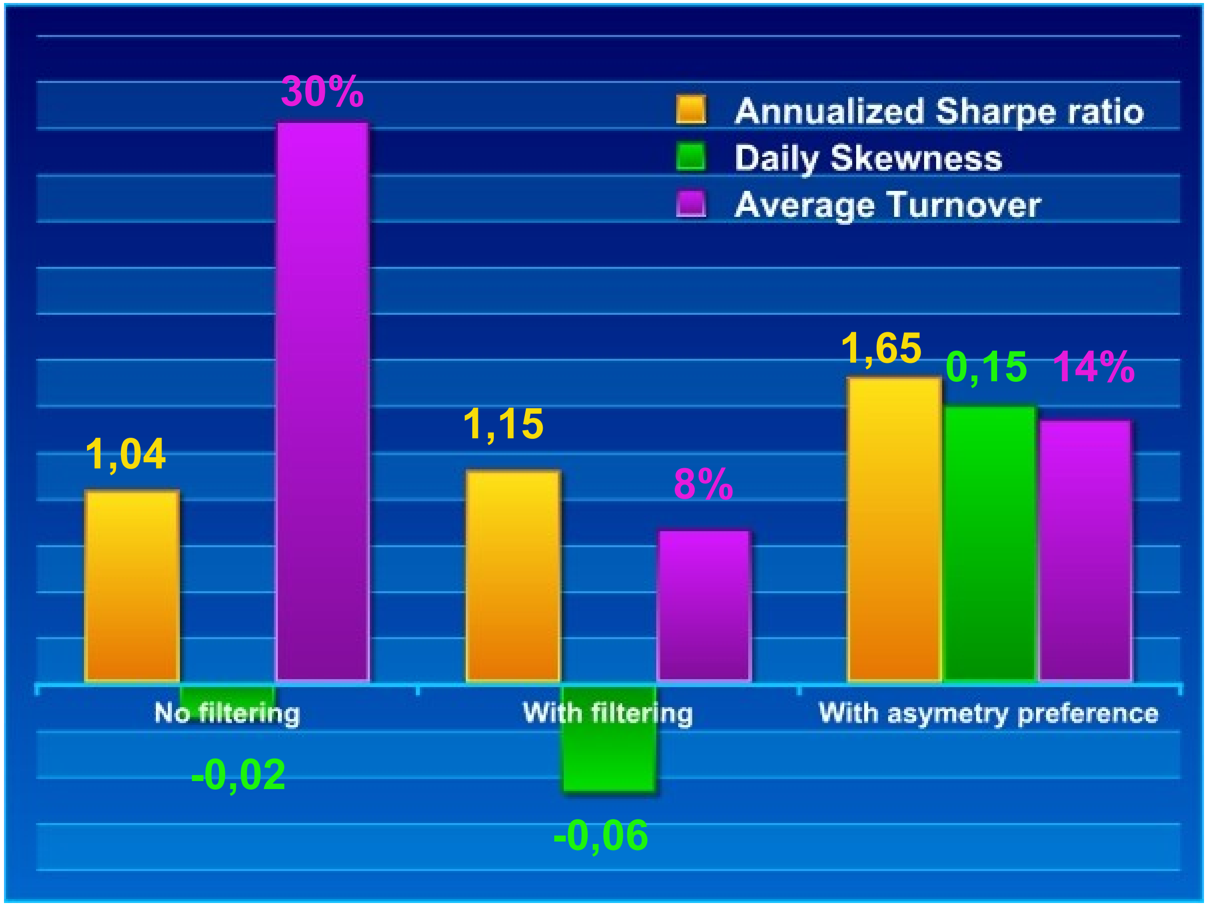
Back-Test : Ex-post Base 100 Performance

Compared return/asymmetry indicators



Compared risk, performance and stability analysis



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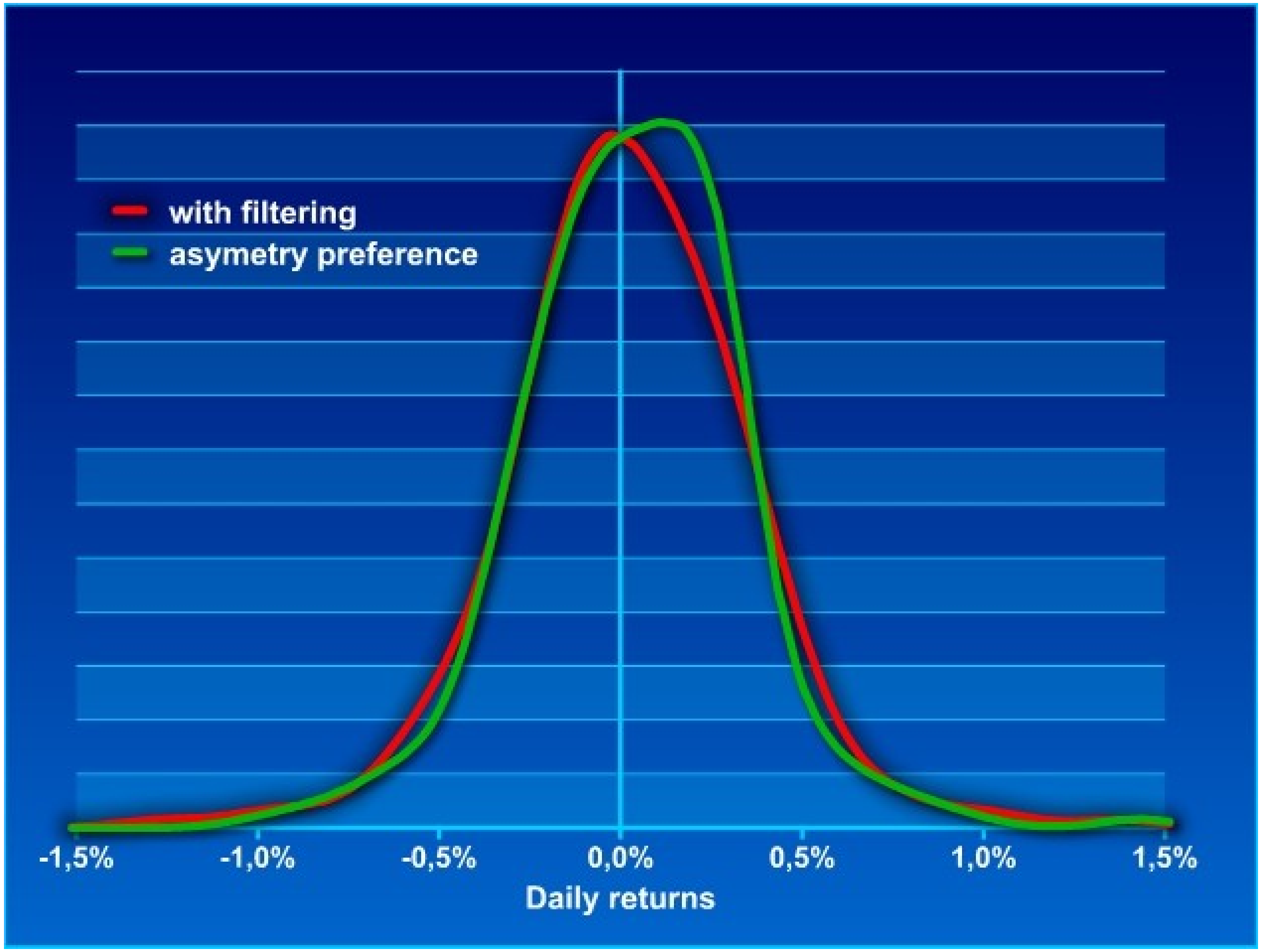
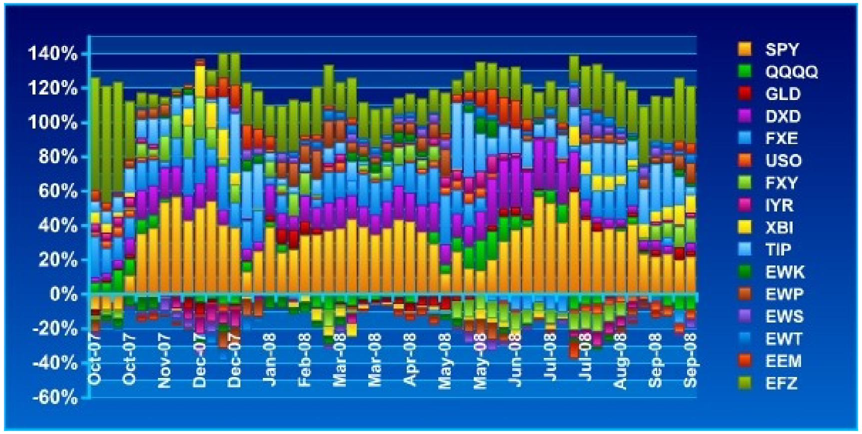
Compared distributions

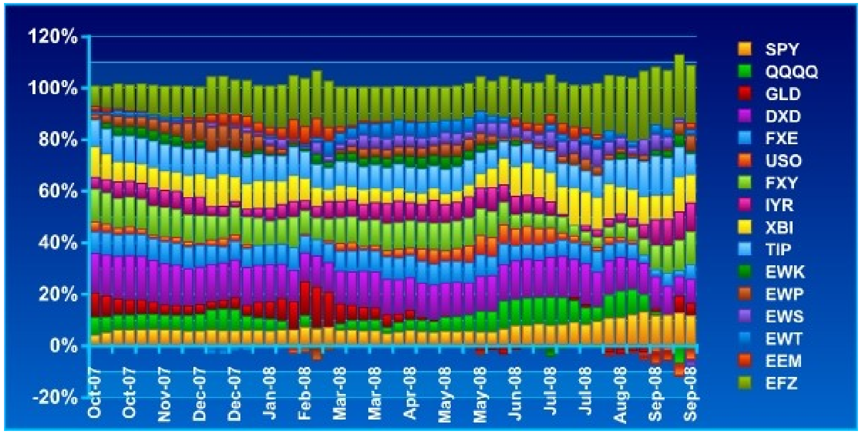




Plain Markowitz

Risk model : empirical covariance matrix

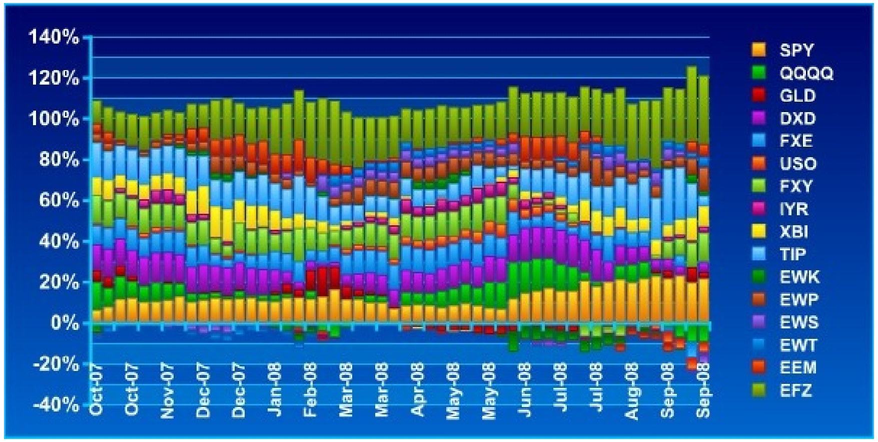
➔Very unstable composition



Filtered Markowitz

Risk model : filtered covariance matrix

➔Very smooth composition



Exotic Convex Optimization Risk model filtered covariance matrix

Allocation model more constrained (asymmetry preferences)

➔ Medium composition stability

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To go further: a real life example