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QUANTITATIVE PORTFOLIO MANAGEMENT

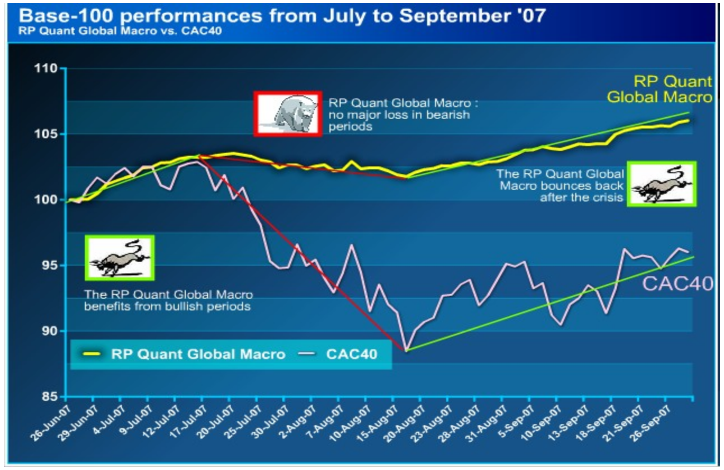
UTS QFRC Occasional Lecture

October 14th, 2008

Convex Optimization

in

Quantitative Finance



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Outline

- Introduction : Optimization in Quant Finance
- Convexity : from Local to Global
- Convexity : Stability Analysis
- Duality for practitionners and convex relaxations of non-convex optimizatoin problems

Optimization in Quantitative Finance

Quantitative teams solve every day optimization problems:

- Optimal Portfolio Construction
- Optimal Calibration/Filtering problems

What kind of certificates/insurance car bring convexity?

What is a reasonable approach when convexity is not there a priori?

Porfolio Optimization

Let $[x_1, x_2, \ldots, x_N]^T \in \mathbb{R}^N$ be the N-vector of portfolio weights,

$\begin{cases} R : \mathbb{R}^N \to \mathbb{R} \\ x \to R(x) \end{cases}$ a convex risk function

$\begin{cases} p : \mathbb{R}^N \to \mathbb{R} \\ x \to p(x) \end{cases}$ a concave performance function,

$\Delta \subset \mathbb{R}^N$ a polyhedral subset of $\mathbb{R}^N$, a target performance level $\overline{p}$

$$\begin{cases} Inf\ R(x) \\ x \in \mathbb{R}^N \\ p(x) \geq \overline{p} \\ x \in \mathbb{R}^N \end{cases}$$

Quadratic Portfolio Optimization

$R(x) := x^T \Gamma x$ with $\Gamma \in S_N^+$ (the cone of positive semidefinite symmetric matrices)

$$p(x) = \rho^T x \text{ where } \rho \in \mathbb{R}^N$$

$$\Delta = \{ y \in \mathbb{R}^N : l_i \leq y_i \leq u_i \; i = 1, \ldots, N \}$$

NB: ρ and Γ might be partially related to first and second order statistical estimators of $(r_{1,} \ldots\ldots, r_N)^T = r$ the N-random variables associated with the returns of the N assets

Convex Risk Function : CVaR

Rockafellar R.T. and S. Uryasev (2000): Optimization of Conditional Value-at-Risk. T*he Journal of Risk*. Vol. 2, No. 3, 2000, 21-41

The $\beta - CVaR$ amounts to minimize over $\theta := (x, s) \in \mathbb{R}^{N+1}$:

$$F(\theta) = R_\beta(x, s) = -s + (1-\beta)^{-1} \int_{r \in \mathbb{R}^N} [-r^T x + s]^+ p(r) \,\mathrm{d}r$$

where: $[t]^+ = max(t, 0)$ and for a fixed portfolio x, variable s can be seen as a « strike » corresponding to a « Put Option » on the fund ex-ante daily returns $r^T x$

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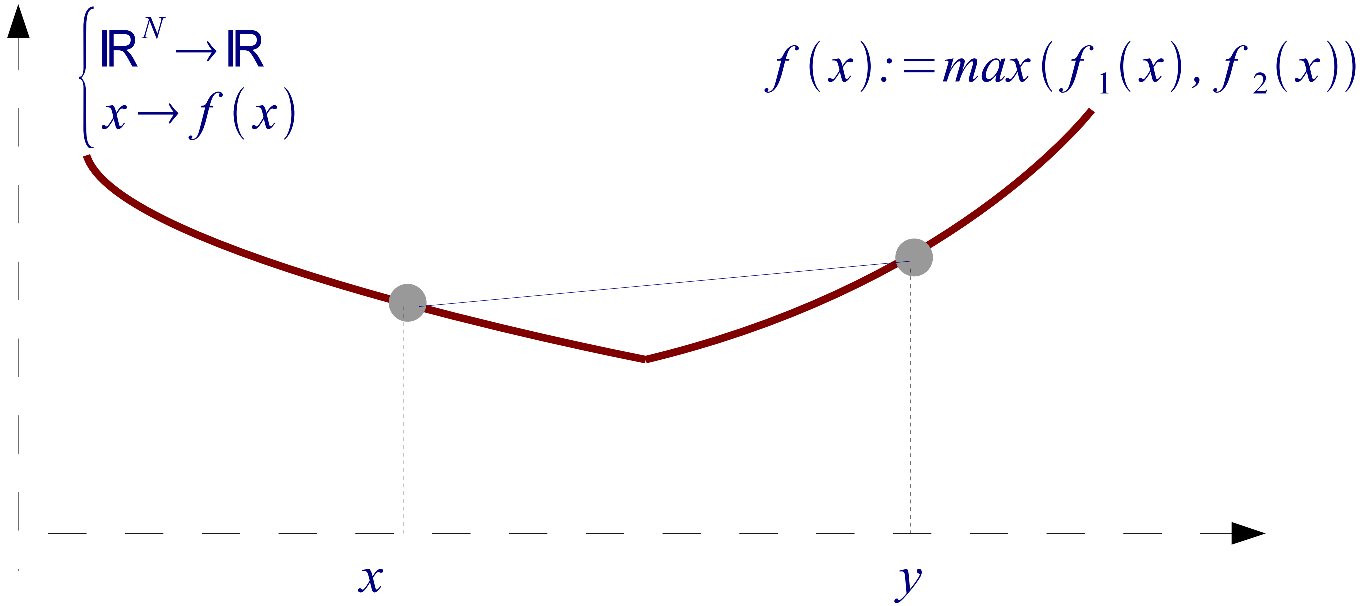
Convex Functions

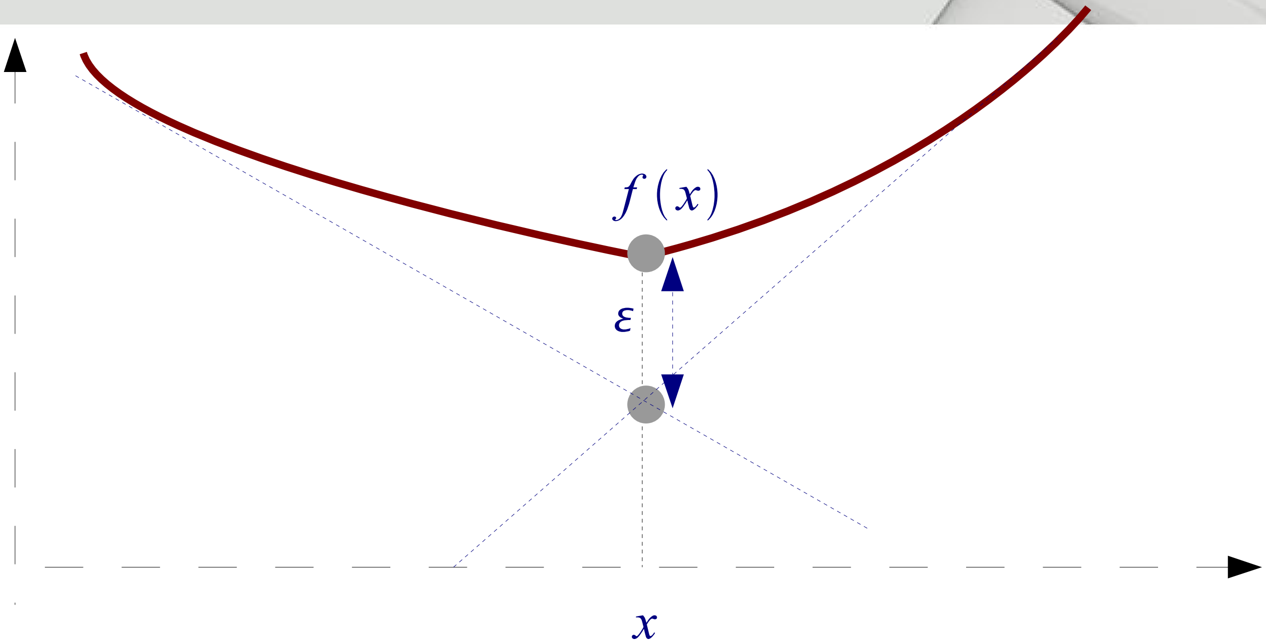


$$\forall (x;y) \in \mathbb{R}^N \times \mathbb{R}^N, \alpha \in [0;1]$$

$$f(\alpha x + (1-\alpha) y) \leqslant \alpha f(x) + (1-\alpha) f(y)$$

Everything derives from this simple property!

ε - Subdifferential : beyond first order analysis

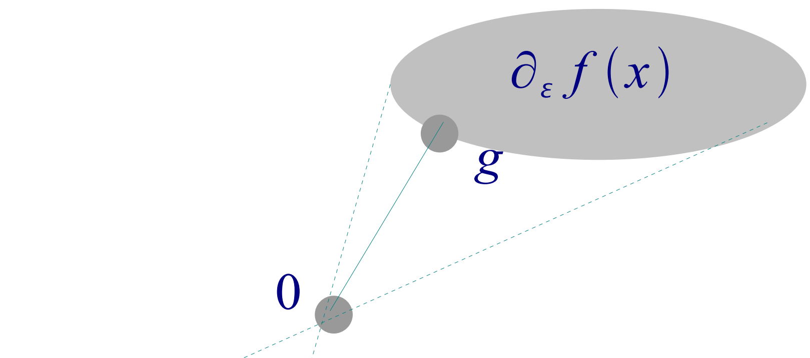


$$\forall x \in \mathbb{R}^N, \varepsilon \in \mathbb{R}_+,$$

$$\partial_\varepsilon f(x) := \{ s \in \mathbb{R}^N : f(y) - f(x) \geqslant < s, y - x > - \varepsilon, \forall y \in \mathbb{R}^N \}$$

ε - Subdifferential : Distance to the Moon

Theorem : $\forall\, x \in \mathbb{R}^N, \varepsilon \in \mathbb{R}_+,\quad \partial_\varepsilon f(x)$ is a compact convex set



Theorem : let, $\varepsilon \geqslant 0 ; \delta \geqslant 0$, and $g \in \partial_\varepsilon f(x)$ such that $\|g\| \leqslant \delta$, then

$$x \; is \; an \,(\varepsilon, \delta)\, minimal \; point \; of \; f$$

$$f(y) \geqslant f(x) - \varepsilon - \delta \|y - x\|, \forall\, y \in \mathbb{R}^N$$

Support function and ε - derivation

Support Function: $\forall d \in \mathbb{R}^N, \varepsilon \in \mathbb{R}_+,$

$$f_\varepsilon'(x,d) := \sigma_{\partial_\varepsilon f(x)}(d)$$

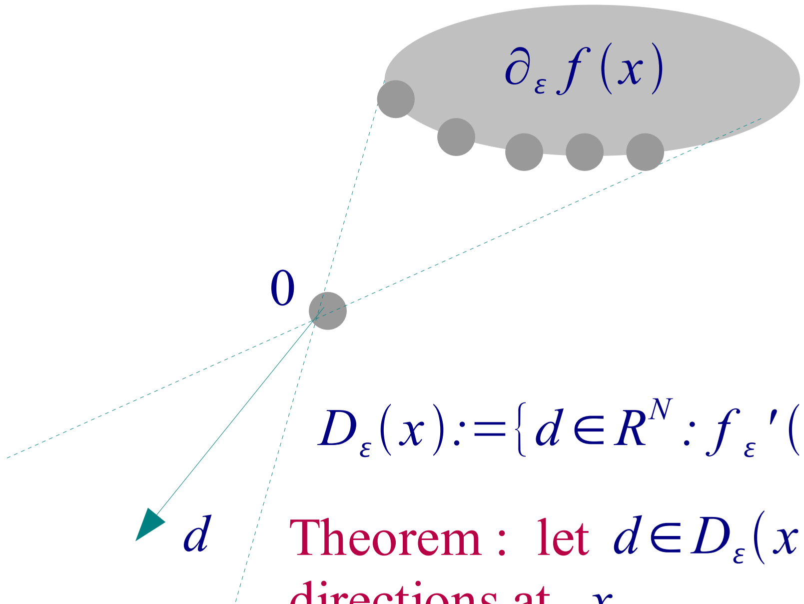
$$:= max_{s \in \partial_\varepsilon f(x)} d^T s$$

ε - Derivation theorem:

$$f_\varepsilon'(x,d) = inf_{t>0} \frac{f(x+td) - f(x) + \varepsilon}{t}$$

Proof: use Fenchel conjugacy Hiriart-Urruty Lemaréchal XI - Theorem 2.1.1

From Local to Global : Visible side of the Moon



$$D_\varepsilon(x) := \{ d \in R^N : f_\varepsilon{}'(x, d) < 0 \}$$

Theorem : let $d \in D_\varepsilon(x)$; then d is an ε - descent directions at x

Proof : there exist $t_\varepsilon > 0$ such that

$$\frac{f(x + t_\varepsilon d) - f(x) + \varepsilon}{t_\varepsilon} < 0 \quad \text{and hence} \quad f(x + t_\varepsilon d) < f(x) - \varepsilon$$

Eigenvalue Optimization in Quant Finance

F. Oustry, *A second order bundle method to minimize the maximum eigenvalue function*, Mathematical Programming, Volume 89, Number 1, 2000

Let S_N be the set of N x N symmetric matrices and $S_N \ni X \rightarrow f(X) := \lambda_{max}(X) \in \mathbb{R}$ the maximum eigenvalue function and define the *spectrahedron* (Nemirovski)

$$S_N^1 := \{ S \in S_N^+ , Trace(S) = 1 \}$$

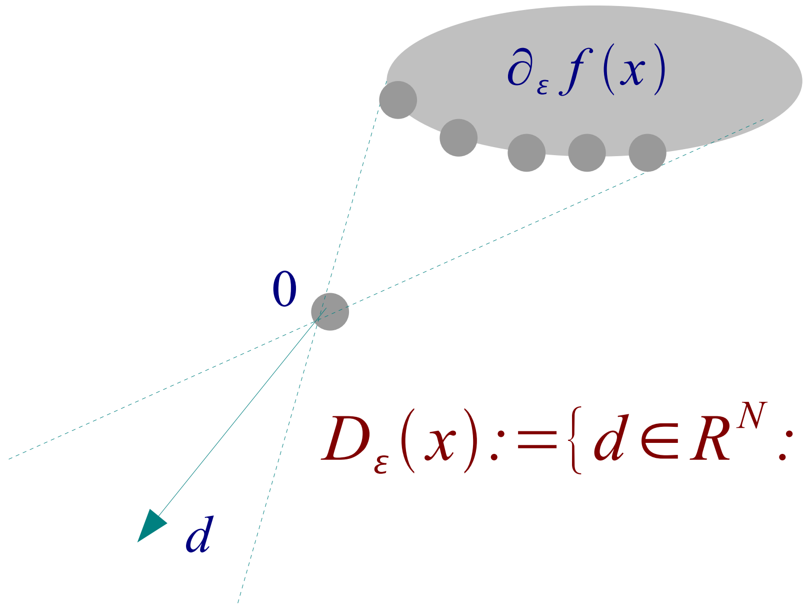
Ky Fan (1949) has shown that not only f is a convex function but it is also the *support function* of S_N^1

$$f(X) = \sigma_{S_N^1}(X) = max\{ < S , X >_F : S \in S_N^1 \}$$

EO : Approximate subdifferential

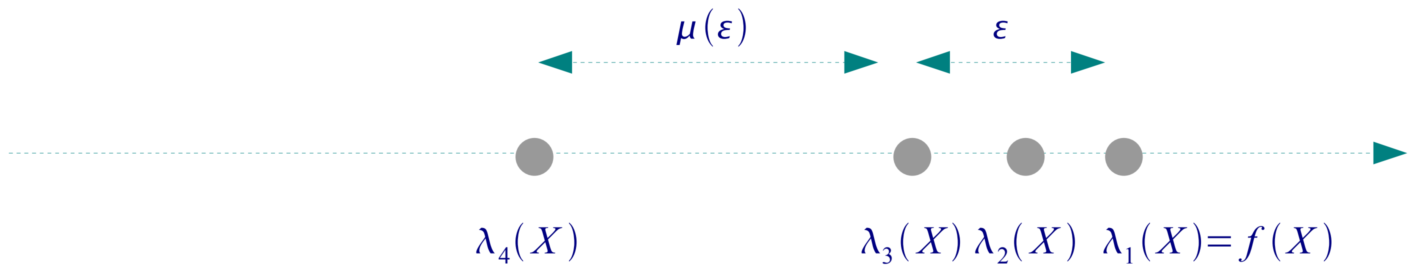
Theorem : $\forall X \in S_N, \varepsilon \geqslant 0,$ we have

$$\partial_\varepsilon f(x) = \{S \in S_N^+, Trace(S) = 1 : < S, X >_F \geqslant f(X) - \varepsilon\}$$



$$D_\varepsilon(x) := \{d \in R^N : f_\varepsilon'(x, d) < 0\}?$$

EO : Approximate subdifferential



Let r_ε be the number of eigenvalues of X non smaller than $f(X)-\varepsilon$ and Q_ε an N x r_ε matrix whose columns form an othonormal basis of the sum of the r_ε first eigenspaces. Then a « good » a good approximation of $\partial_\varepsilon f(X)$ is: $\exists \mu(\varepsilon)>0$

$$\partial_{\mu(\varepsilon)} f(X) \subset \delta_\varepsilon f(X) := Q_\varepsilon S^1_{r_\varepsilon} Q_\varepsilon^T \subset \partial_\varepsilon f(X)$$

$$f_{\mu(\varepsilon)}'(X;.) \leq \sigma_{\delta_\varepsilon f(X)}(.) \leq f_\varepsilon'(X;.)$$

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EO : Random Matrices

L. Laloux, P. Cizeau, J-P. Bouchaud and M. Potters, Noise Dressing of Financial Correlation Matrices. *Phys. Rev. Lett.* 83, 1467 - 1470 , 1999.

Random Matrix theory[8,10]. We will note $\rho_C(\lambda)$ the density of eigenvalues of $\mathbf{C}$, defined as:

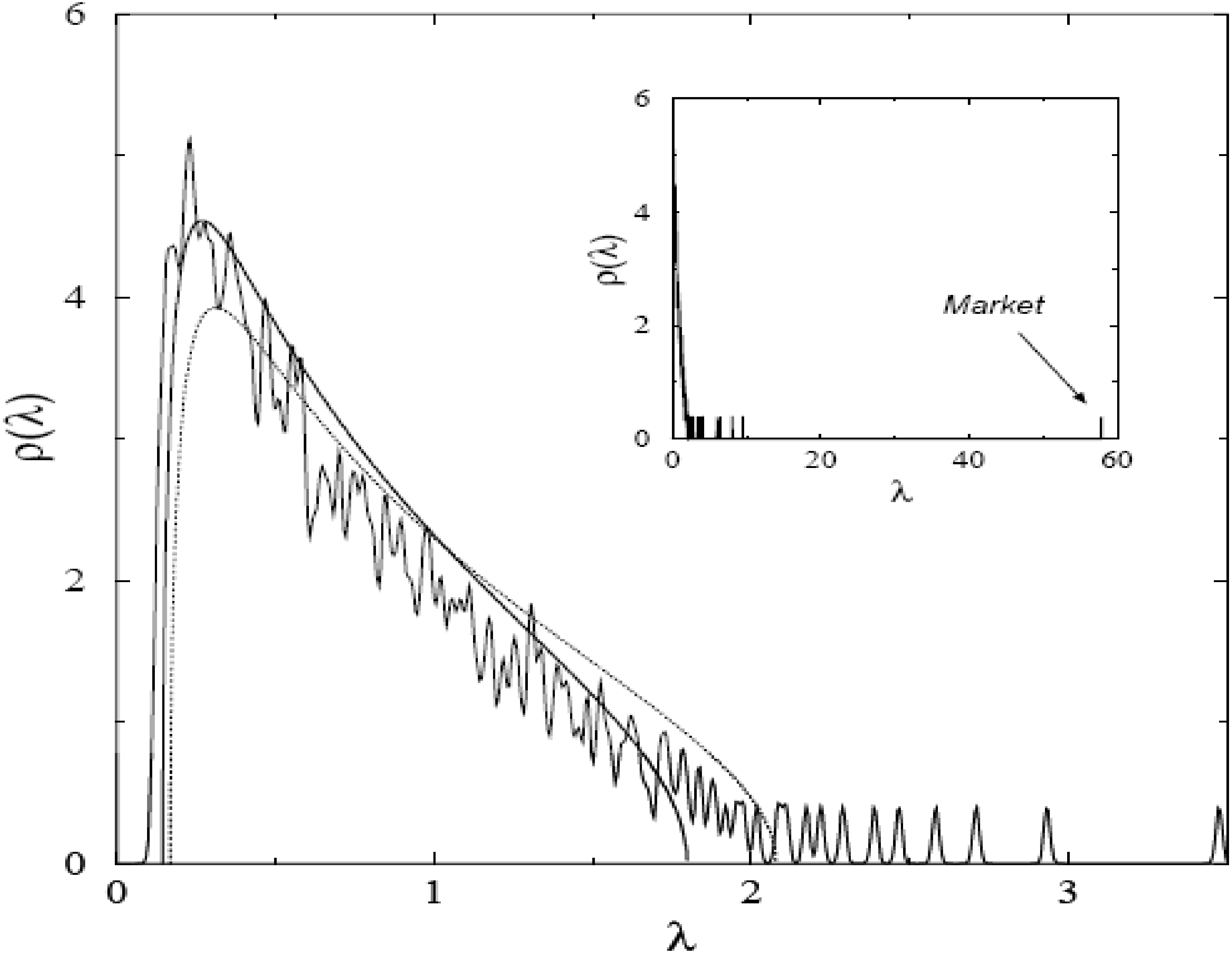
$$\rho_C(\lambda) = \frac{1}{N}\frac{dn(\lambda)}{d\lambda}, \tag{0.2}$$

where $n(\lambda)$ is the number of eigenvalues of $\mathbf{C}$ less than λ.

Interestingly, if $\mathbf{M}$ is a $T \times N$ random matrix, $\rho_C(\lambda)$ is self-averaging and exactly known in the limit $N \to \infty$, $T \to \infty$ and $Q = T/N \geq 1$ fixed [8,9], and reads:

$$\begin{aligned} \rho_C(\lambda) &= \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max}-\lambda)(\lambda-\lambda_{min})}}{\lambda}, \\ \lambda_{min}^{max} &= \sigma^2(1 + 1/Q \pm 2\sqrt{1/Q}), \end{aligned} \tag{0.3}$$

with $\lambda \in [\lambda_{min}, \lambda_{max}]$, and where σ^2 is equal to the variance of the elements of $\mathbf{M}$ [9], equal to 1 with our normalisation. In the limit $Q = 1$ the normalised eigenvalue



6
4
2
0
0
1
2
3
ρ(λ)
λ
6
4
2
0
0
20
40
60
ρ(λ)
λ
Market

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EO : Covariance Matrices Filtering

Let $\overline{C}$ be a target symmetric matrix, we want to find $C \in S_N$ such that:

$$\begin{cases} Min \left\| C - \overline{C} \right\|^2 \\ C - \sigma^2 I_N \in S_N^+ \\ C_{ii} = \sigma_i^2 , i = 1 ; \ldots\ldots , N \end{cases} \Leftrightarrow \begin{cases} Min \left\| C - \overline{C} \right\|^2 \\ \lambda_{max} (\sigma^2 I_N - C) \leq 0 \\ C_{ii} = \sigma_i^2 , i = 1 ; \ldots\ldots , N \end{cases}$$

CVaR : subdifferential

$$F(\theta) = \mathsf{E}\{f(\theta)\} = \int_{\Omega} f(\theta, \omega) P(d\omega)$$

where $f(\theta)$ is a measurable random function (whose realization is denoted by $f(\theta, \omega)$ as in the above integral) defined on a probability space $(\Omega, \mathcal{F}, P)$. Let the domain of f be a convex open set $D \subset \mathbf{R}^m$ and suppose that, for every $\theta \in D$, $\mathsf{E}\{|f(\theta)|\} < \infty$. We now give sufficient conditions for directional differentiability of F at a point $\theta_0 \in D$ (cf. [10]).

Assumption 2.1: There exists a positive-valued random variable $K = K(\omega)$ such that $\mathsf{E}\{K\}$ is finite and

$$|f(\theta_1, \omega) - f(\theta_2, \omega)| \leq K(\omega) \|\theta_1 - \theta_2\| \qquad (2.2)$$

for almost all $\omega \in \Omega$ and for all $\theta_1, \theta_2 \in D$.

Assumption 2.2: With probability one (w.p.1) the function $f(\theta)$ is directionally differentiable at θ_0.

CVaR : subdifferential

Suppose now that, in addition to the assumptions of Proposition 2.1, the function f is subdifferentiable at the point θ_0 w.p.1. Of course, in the convex case, this subdifferentiability follows from convexity of f. Then formula (2.3) implies that $F'(\theta, d)$ is convex in d and hence F is also subdifferentiable at θ_0. Moreover

$$\partial F(\theta_0) = \mathbb{E}\{\partial f(\theta_0)\} = \int_\Omega \partial f(\theta_0, \omega) P(d\omega) \qquad (2.4)$$

Shapiro, A. and Wardi, Y., *Nondifferentiability of the Steady-State Function in Discrete Event Dynamic Systems*, IEEE Transactions on Automatic Control, vol. 39, pp. 1707-1711, 1994.

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The U-Lagrangian of a convex function

Lemaréchal, F. Oustry, and C. Sagastizábal, *The U-Lagrangian of a convex function*, Trans. Amer. Math. Soc., vol. 352, no. 2, pp. 711-729, 2000

Definition 2.1.

(*i*) Define $\mathcal{U}_1$ as the subspace where $f'(\bar{p};\cdot)$ is linear and take $\mathcal{V}_1 := \mathcal{U}_1^\perp$. Because $f'(\bar{p};\cdot)$ is sublinear, we have

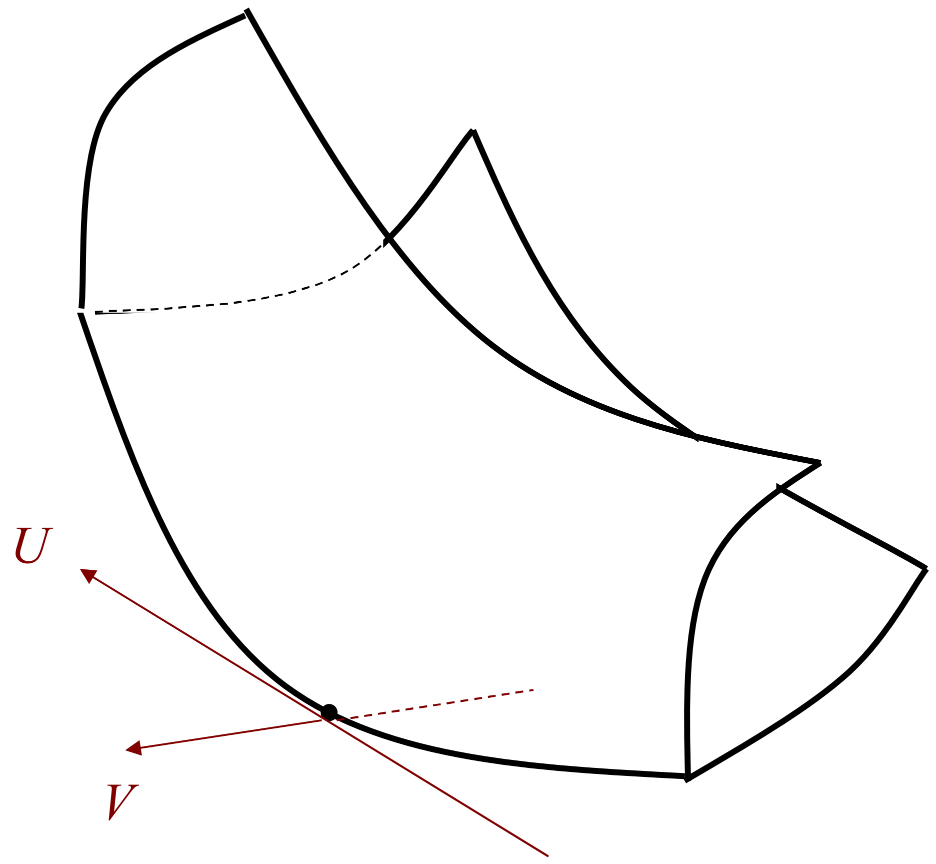
$$\mathcal{U}_1 := \{d \in \mathbb{R}^n : f'(\bar{p};d) = -f'(\bar{p};-d)\}\,;$$

if necessary, see for instance Proposition V.1.1.6 in [11]. In other words, $\mathcal{U}_1$ is the subspace where $f(\bar{p}+\cdot)$ appears to be "differentiable" at 0. Note that this definition of $\mathcal{U}_1$ does not rely on a particular scalar product.

(*ii*) Define $\mathcal{V}_2$ as the subspace parallel to the affine hull of $\partial f(\bar{p})$ and take $\mathcal{U}_2 := \mathcal{V}_2^\perp$. In other words, $\mathcal{V}_2 := \operatorname{lin}(\partial f(\bar{p}) - \bar{g})$ for an arbitrary $\bar{g} \in \partial f(\bar{p})$, and $d \in \mathcal{U}_2$ means $\langle \bar{g}+v, d\rangle = \langle \bar{g}, d\rangle$ for all $v \in \mathcal{V}_2$.

(*iii*) Define $\mathcal{U}_3$ and $\mathcal{V}_3$ respectively as the normal and tangent cones to $\partial f(\bar{p})$ at an arbitrary g° in the relative interior of $\partial f(\bar{p})$. It is known (see for example Proposition 2.2 in [14]) that the property $g^\circ \in \operatorname{ri} \partial f(\bar{p})$ is equivalent to these cones being subspaces. $\square$

The U-Lagrangian of a convex function



The U-Lagrangian of a convex function

3.1. **Definition and basic properties.** Following the above introduction, we define the function $L_{\mathcal{U}}$ as follows:

$$\mathcal{U} \ni u \mapsto L_{\mathcal{U}}(u) := \inf_{v \in \mathcal{V}} \left\{ f(\bar{p} + u \oplus v) - \langle \bar{g}_{\mathcal{V}}, v \rangle_{\mathcal{V}} \right\}. \tag{3.1}$$

Associated with (3.1) we have the set of minimizers

$$W(u) := \operatorname*{Argmin}_{v \in \mathcal{V}} \left\{ f(\bar{p} + u \oplus v) - \langle \bar{g}_{\mathcal{V}}, v \rangle_{\mathcal{V}} \right\}. \tag{3.2}$$

It will be seen below that an important question is whether $W(u)$ is nonempty.

In particular, $L_{\mathcal{U}}$ is differentiable at 0, with $\nabla L_{\mathcal{U}}(0) = \bar{g}_{\mathcal{U}}$.

Second Order Expansion and Stability

Theorem 3.9. *Take $\bar{g} \in \operatorname{ri} \partial f(\bar{p})$ and let the $\mathcal{U}$-Hessian $\mathrm{H}_{\mathcal{U}} f(\bar{p})$ exist. For $u \in \mathcal{U}$ and $h \in u \oplus W(u)$, there holds*

$$f(\bar{p}+h) = f(\bar{p}) + \langle \bar{g}, h \rangle + \frac{1}{2} \langle \mathrm{H}_{\mathcal{U}} f(\bar{p}) u, u \rangle_{\mathcal{U}} + o(||h||^2) . \tag{3.10}$$

Assume now that f depends on uncertainties θ, $f = f(p, \theta)$, with $0 \in ri \partial f(\overline{p}, 0)$ and $H_U f(\overline{p}, 0)$ exists and is positive semidefinite. We also assume that $\theta \rightarrow f(p, \theta)$ is continuously differentiable.

Then in a neighborhood of $\theta = 0$, the optimal solution $p(\theta)$ is locally Lipschitz continuous.

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Lagrangian Duality

Consider first an optimization problem put in the form

$$\inf f(x)\,, \quad x \in \mathcal{X}\,, \quad g_j(x) = 0,\ j = 1, \dots, m\,. \tag{1}$$

We introduce the *Lagrangian*, a function of the primal variable x and of the *dual variable* – or *multipliers* – $u \in \mathbb{R}^m$:

$$\mathcal{X} \times \mathbb{R}^m \ni (x, u) \mapsto L(x, u) := f(x) + \sum_{j=1}^{m} u_j g_j(x) = f(x) + u^\top g(x)\,, \tag{2}$$

if $g \in \mathbb{R}^m$ denotes the vector of constraint-values. The *dual function* is then a function of u alone, defined by

$$\mathbb{R}^m \ni u \mapsto \theta(u) := \inf_{x \in \mathcal{X}} L(x, u)\,, \tag{3}$$

and the *dual problem* is

$$\sup \theta(u)\,, \quad u \in \mathbb{R}^m\,. \tag{4}$$

If some of the constraints in (1) are inequalities $g_j(x) \leqslant 0$, then the correspon-ding dual variables have a sign-constraint: $u_j \geqslant 0$. The dual function is always concave and upper semi-continuous, *independently* of the data $\mathcal{X}$, f and g.

Semidefinite Duality

A convenient scalar product on our constraint-space $\mathcal{S}_n$ is the ordinary dot-product in $\mathbb{R}^{n^2}$:

$$(\mathcal{S}_n \times \mathcal{S}_n) \ni (X, U) \mapsto \langle X, U \rangle := \operatorname{tr} XU = \sum_{i,j=1}^{n} X_{ij} U_{ij} \,,$$

also called the trace-product. The reason it is so convenient is the following relation: for all $X \in \mathcal{S}_n$ and $u \in \mathbb{R}^n$,

$$[\textstyle\sum_{i,j=1}^{n} X_{ij} u_i u_j =]\ u^\top X u = \langle X, uu^\top \rangle \,, \tag{11}$$

which can be checked in a straightforward way (note: $(uu^\top)_{ij} = u_i u_j$); it will be used continually.

Semidefinite Duality

$$\inf f(x) := b^\top x\,, \quad x \in \mathcal{X} := \mathbb{R}^m\,, \quad g(x) := A_0 + \sum_{j=1}^{m} x_j A_j \succcurlyeq 0\,, \tag{13}$$

where $b \in \mathbb{R}^m$ is given, as well as each A_j in $\mathcal{S}_n$. Writing the Lagrangian dual (10) of this conic problem is an easy exercise:

– Form the Lagrangian at $(x, U) \in \mathbb{R}^m \times K^\circ = \mathbb{R}^m \times \mathcal{S}_n^-$

$$L(x, U) = b^\top x + \langle g(x), U \rangle = \sum_{j=1}^{m} (b_j + \langle A_j, U \rangle) x_j + \langle A_0, U \rangle\,, \tag{14}$$

– Calculate the dual function: for $U \succcurlyeq 0$,

$$\theta(U) = \begin{cases} \langle A_0, U \rangle & \text{if } b_j + \langle A_j, U \rangle = 0 \text{ for } j = 1, \ldots, m\,, \\ -\infty & \text{otherwise}\,. \end{cases}$$

Dualizing a quadratic program

Let be given $m+1$ quadratic functions defined on $\mathbb{R}^n$:

$$q_j(x) := x^\top Q_j x + b_j{}^\top x + c_j\,,\ j = 0, \ldots, m\,,$$

where the Q_j's lie in $\mathcal{S}_n$, the b_j's in $\mathbb{R}^n$ and the c_j's in $\mathbb{R}$; we assume $c_0 = 0$. With these data, consider the quadratic problem

$$\begin{cases} \inf q_0(x)\,, & x \in \mathbb{R}^n \\ q_j(x) = 0,\ j = 1, \ldots, m\,. \end{cases} \tag{21}$$

The associated Lagrangian (2) of §2.1 with $\mathcal{X} := \mathbb{R}^n$, $f := q_0$ and $g_j := q_j$, $j = 1, \ldots, m$ is then

$$L(x, u) = x^\top Q(u) x + b(u)^\top x + c(u)\,, \tag{22}$$

where $Q(u) := Q_0 + \sum_{j=1}^m u_j Q_j$, $b(u) := b_0 + \sum_{j=1}^m u_j b_j$ and $c(u) = \sum_{j=1}^m u_j c_j = c^\top u$. Just as in §2.1, we denote by

$$\mathbb{R}^m \ni u \mapsto \theta(u) := \inf_{x \in \mathbb{R}^n} L(x, u) \tag{23}$$

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The Dual of a Quadratic Program is SDP!!

Proposition 4.1

$$\theta(u) = \begin{cases} c(u) - \frac{1}{4} b(u)^\top Q(u)^\dagger b(u) & \textit{if } Q(u) \succcurlyeq 0 \textit{ and } b(u) \in \mathcal{R}(Q(u)), \\ -\infty & \textit{otherwise.} \end{cases}$$

Proof. Just apply Lemma 3.6 and its extension Lemma 3.7. □

Then Schur's Lemma enables a "PD-representation" of the epigraph of the dual function (23) (terminology appearing in [15, § 6.4.3]), and thereby an SDP-formulation of the dual problem:

Corollary 4.2 *The dual of* (21), (22) *is equivalent to the SDP problem with variables* $u \in \mathbb{R}^m$ *and* $r \in \mathbb{R}$*:*

$$\begin{cases} \sup r, \\ \begin{bmatrix} c(u) - r & \frac{1}{2} b(u)^\top \\ \frac{1}{2} b(u) & Q(u) \end{bmatrix} \succcurlyeq 0. \end{cases} \tag{24}$$

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Bidualization and lifting procedure

Theorem 4.4 *The dual of (24), i.e., the bidual of (21), is*

$$(SDP) \qquad \begin{cases} \inf \langle Q_0, X\rangle + b_0^\top x\,, \quad X \in \mathcal{S}_n,\ x \in \mathbb{R}^n\,, \\ \langle Q_j, X\rangle + b_j^\top x + c_j = 0,\ j = 1, \ldots, m\,, \\ \begin{bmatrix} 1 & x^\top \\ x & X \end{bmatrix} \succcurlyeq 0\,. \end{cases}$$

$$\Leftrightarrow$$

$$\begin{cases} \inf \langle Q_0, X\rangle + b_0^\top x\,, \quad X \in \mathcal{S}_n,\ x \in \mathbb{R}^n\,, \\ \langle Q_j, X\rangle + b_j^\top x + c_j = 0,\ j = 1, \ldots, m\,; \\ X \succcurlyeq xx^\top\,. \end{cases}$$

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Portfolio Optimization with third order constraint

$$
\begin{cases}
\min\limits_{\omega} \ \omega^T \Gamma \omega \\
s.t. \\
\qquad \sum\limits_{i,j,k} \omega_i \omega_j \omega_k H_{ijk} \geq h \\
\qquad \rho^T \omega \geq \ell \\
\qquad \omega \in \Delta
\end{cases}
\Leftrightarrow
\begin{cases}
\min\limits_{\omega} \ \omega^T \Gamma \omega \\
s.t. \\
\qquad \left\langle \sum\limits_{k} \omega_k H_{[k]}, W \right\rangle \geq h \\
\qquad \rho^T \omega \geq \ell \\
\qquad \omega \in \Delta \\
\qquad W = \omega \omega^T
\end{cases}
$$

$$\text{where } H_{[k]} \text{ is the } N \times N \text{ matrix whose } i, j \text{ element is } H_{ijk}$$

[1] E. Jondeau and M. Rockinger, Optimal Portfolio Allocation Under Higher Moments, *Working Paper N. 108*, December 2002 (Revised: January 2004). Available at http://www.banque-france.fr/gb/publications/ner/1-108.htm

[1] C.R. Harvey, M. Liechty, J. Liechty and P. Müller. Portfolio Selection with Higher Moments. (December 13, 2004). Available at SSRN: http://ssrn.com/abstract=634141

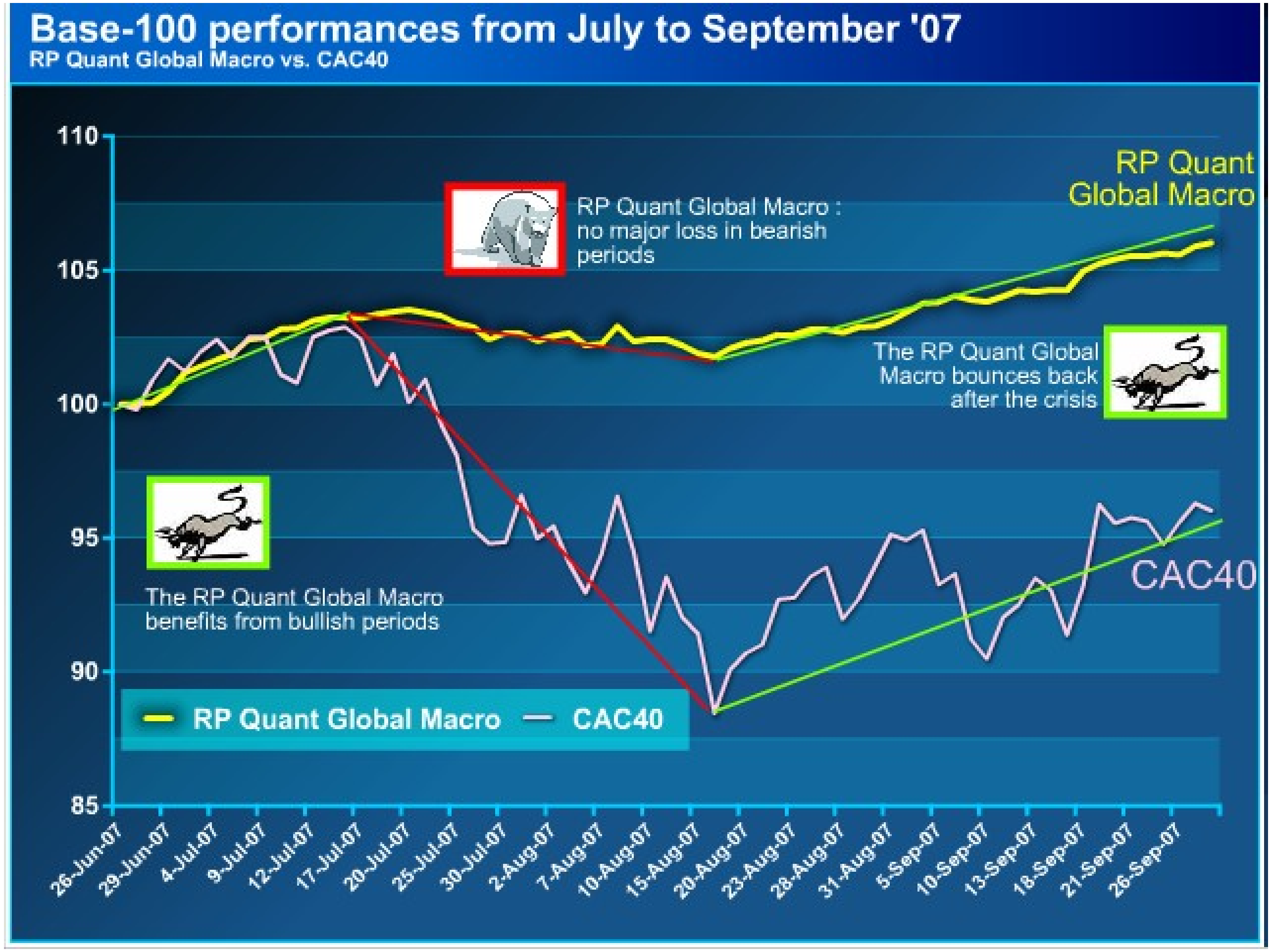
Second Lifting Procedure

$$
\begin{cases}
\min\limits_{\omega} \ \langle \Gamma, W \rangle \\
s.t. \\
\qquad \langle H, Z \rangle \geq h \\
\qquad Z \succeq (W, \omega) \otimes (W, \omega)^{*} \\
\qquad \rho^{T} \omega \geq \ell \\
\qquad \omega \in \Delta \\
\qquad W \succeq \omega \omega^{T}
\end{cases}
$$

$$\textit{where } Z_{ijk} \textit{ is the } N \times N \times N \textit{ third order portolio tensor}$$

[1] J. B. Lasserre, Global Optimization with Polynomials and the Problem of Moments, *SIAM Journal on Optimization*, Vol. 11, No. 3, pp. 796–817, 2001.

Creating Positive Aymmetry



Diversified (GM) Investment Universe

From “Plain Markowitz” to “Exotic Convex Optimization”

- **Plain Markowitz**
  - Performance model : « trend following »
  - Risk model : empirical covariance matrix
  - Allocation model : Minimize Volatility s.t. Perf constraint

- **Filtered Markowitz**
  - Performance model : « trend following »
  - Risk model : filtered covariance matrix
  - Allocation model : Minimize Volatility s.t. Perf constraint

- **Exotic Convex Optimization**
  - Performance model : « trend following »
  - Risk Model : Filtered covariance matrix + Higher moments/CVaR …
  - Allocation model : Minimize Volatility s.t. Perf constraint & Asymmetry preferences (third order moment, CVaR constraints, …)

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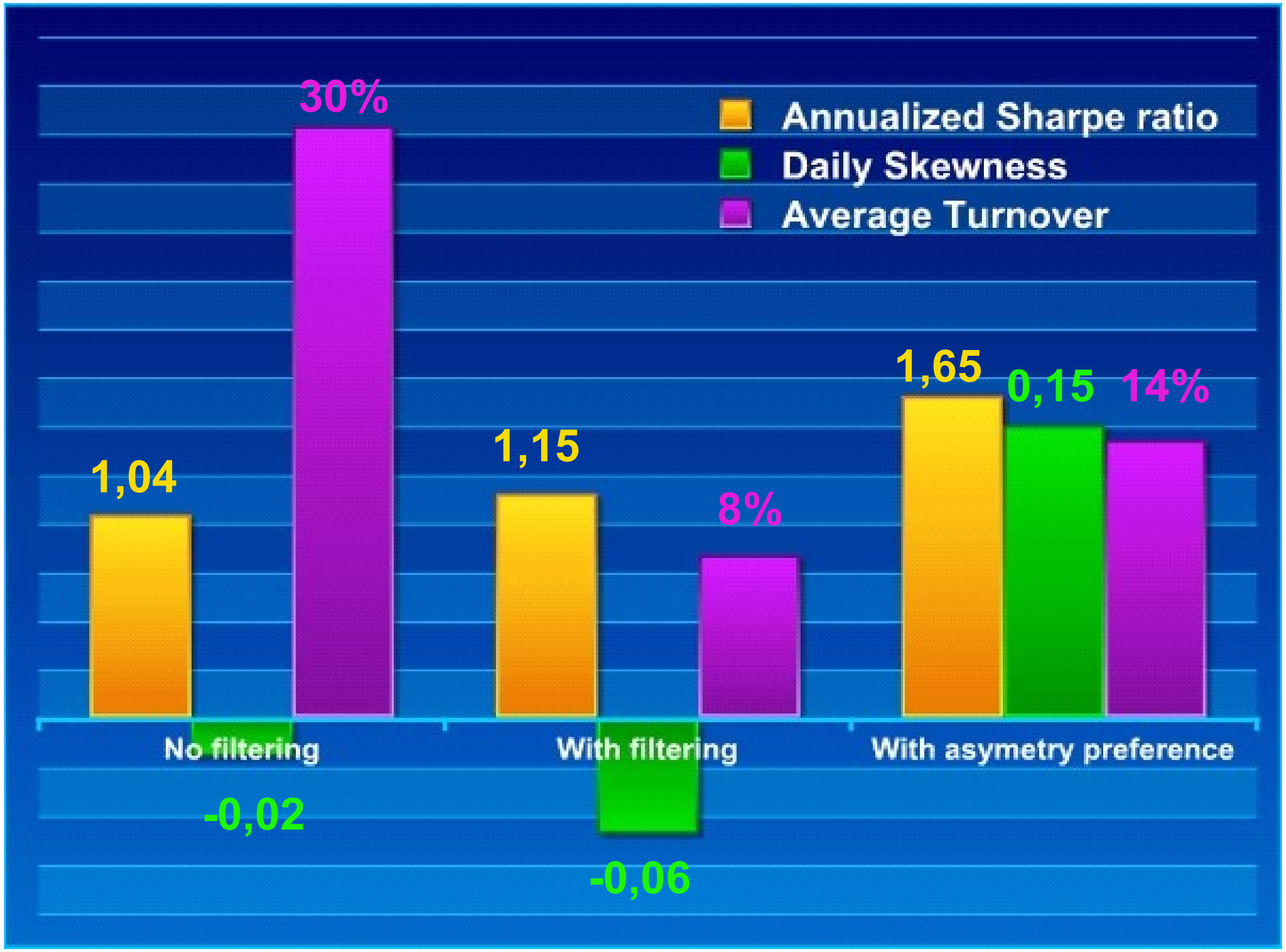
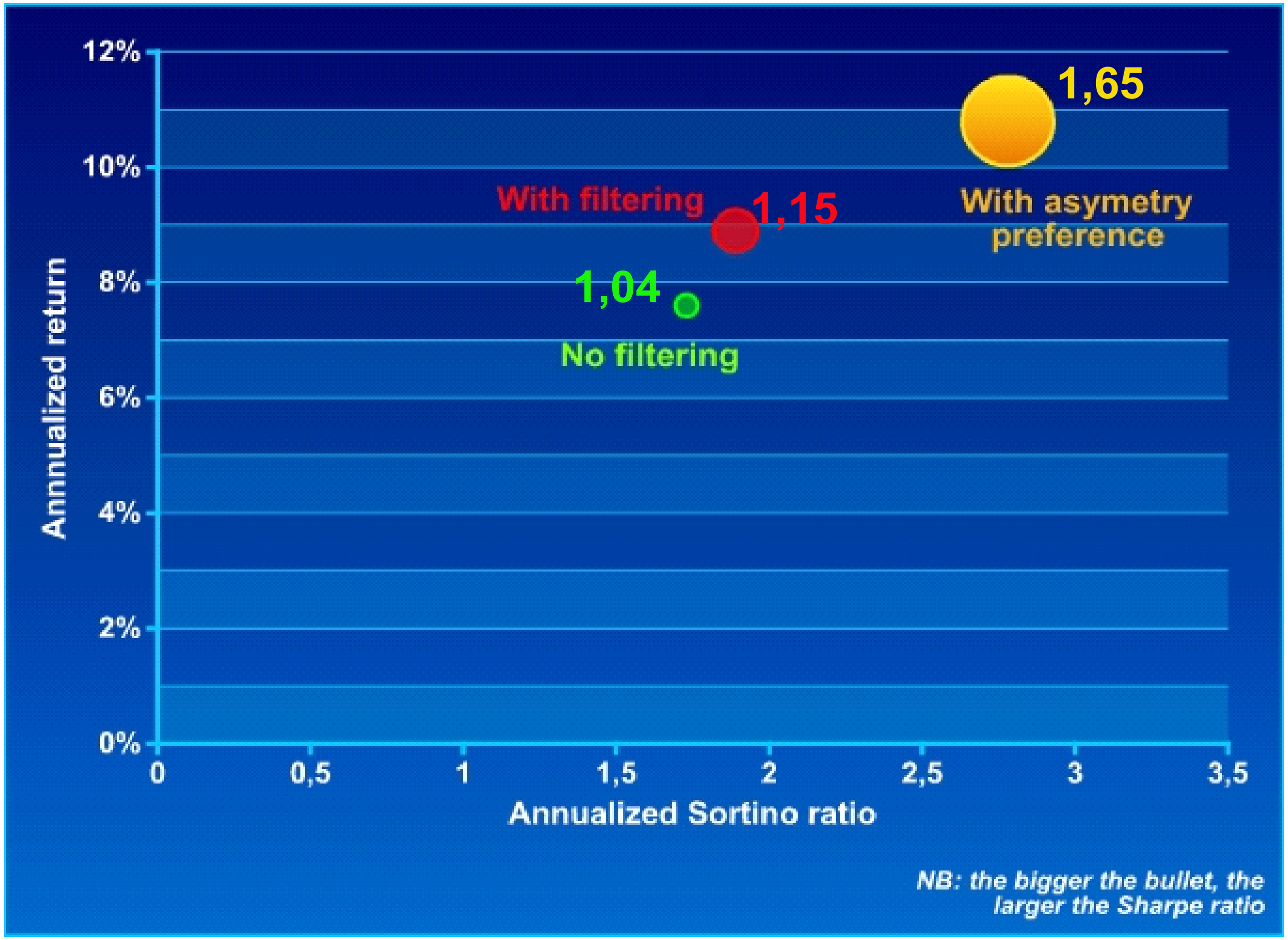
Preference for positive asymmetry in practice



Preference for positive asymmetry in practice



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To go further: a real life example