

June 2008

An introduction to the traded Inflation Market – A Risk Manager's Perspective

Steve McCarthy

Risk types in the Inflation market

- This talk will focus on two major risks
 - Market Risk
 - Risk of losing money due to adverse movements in the market
 - Risk of mis-pricing into the market due to inflation estimation errors
 - Risk of mis-pricing due to model mis-specification
 - Credit Risk
 - Risk of losing money due to counterparty default

Major risk types in the market

Section 1 - Market Risk

Linker Bond Market

- Inflation Payers
 - Commonwealth and State Governments
 - Utilities eg Electricity, ...
 - Property Trusts
 - Project Finance
- Inflation Receivers
 - Pension Funds
 - Insurance Companies
- Inflation Receivers / Payers
 - Investment Banks
 - Hedge Funds
 - Proprietary desks
- Newer Players (Other asset classes - affected by inflation)
 - People wishing to hedge against the Commodity price movements
 - People wishing to hedge against deteriorating Earnings in the share market

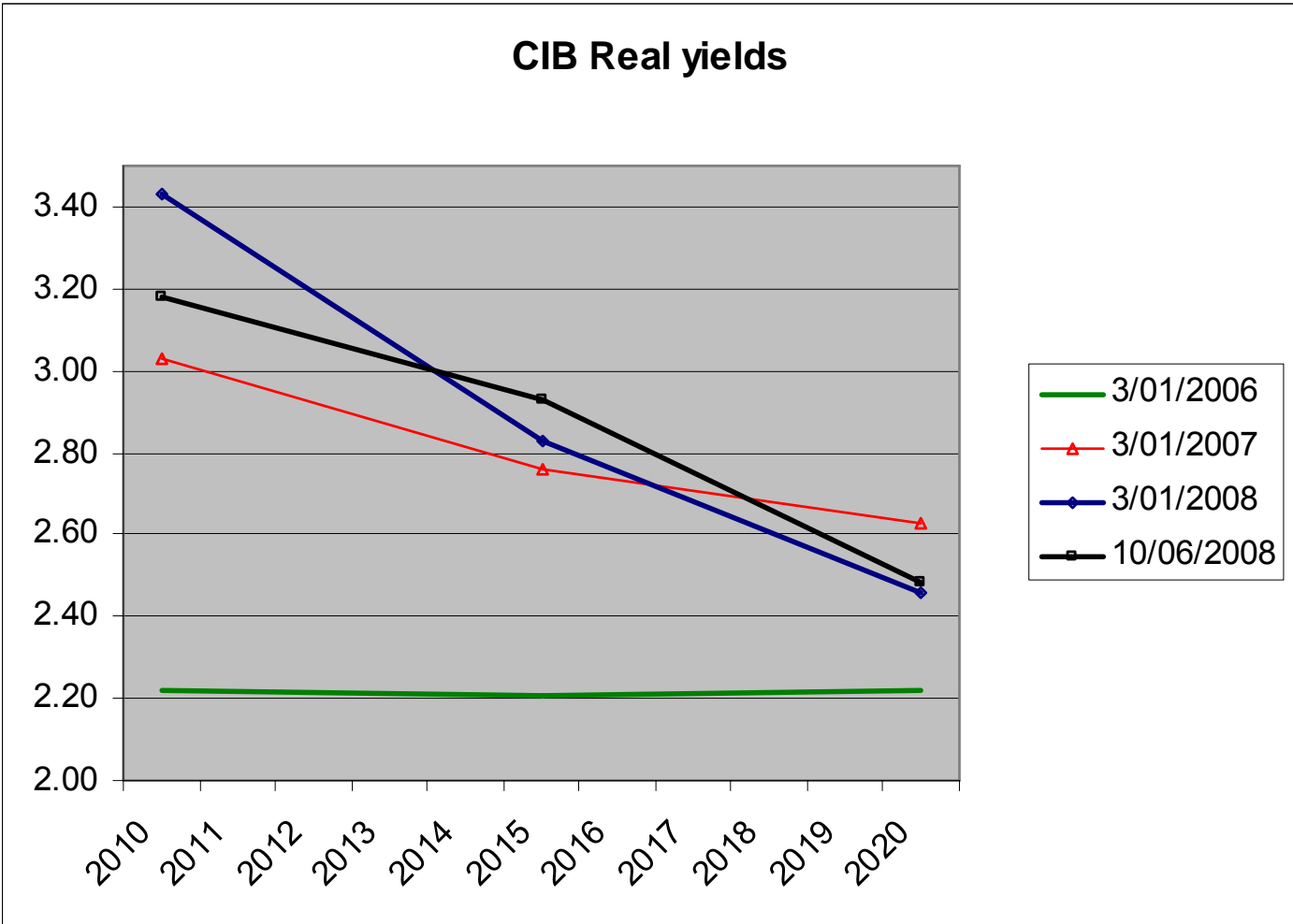
The Inflation market is becoming a core market in itself rather than a sub-component of the bond market

AUD Linker Bond Market

- Composed of multiple bond types
 - Capital Indexed Bonds (CIB)
 - 3 issues by Commonwealth Government
 - Maturing 20/8/2010, 20/8/2015, 20/8/2020
 - Some Semi-Government issues
 - Mostly Corporate issues
 - Longest maturity (about 20 years)
 - Index Annuity Bonds (IAB)
 - Issued mainly by Semi-Government
 - Some Corporate issues
 - Longest maturity (about 25 years)

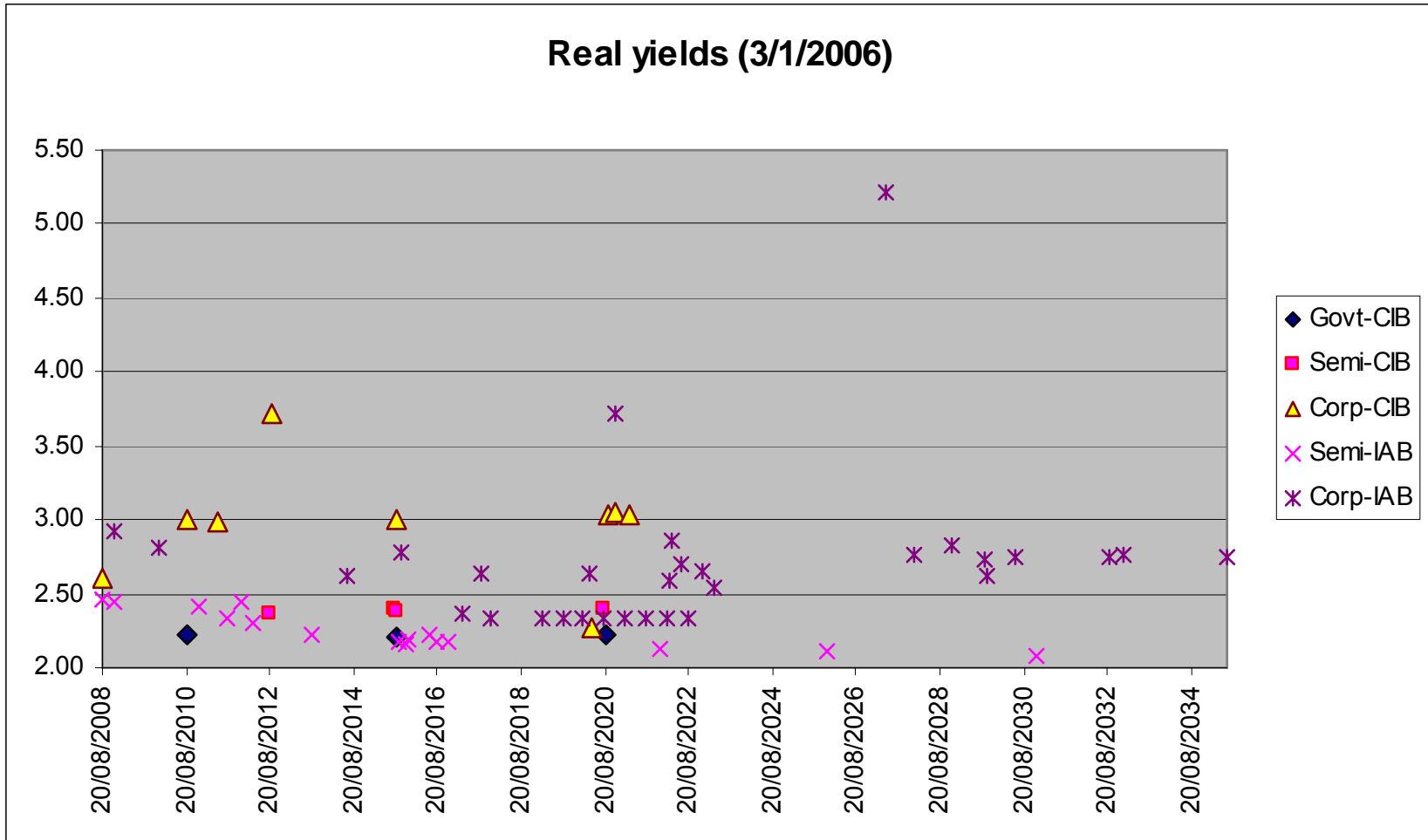
Major issuers in the Market

Linker Bond Market



The longer dated yields seem to be distorted

Linker Bond Market



Long dated bootstrapping requires either considerable extrapolation of the risk-free curve or managing the multitude of different credit types.

ICAP officially released their broker page at the beginning of 2008

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07:52 06JUN08      AUST INFLATION SWAP INTERBANK AVERAGE AU18992      CPISWAPREF
SWAP VALUATION RATES AT 16:30 5/6/08      3YR 93.23      10YR 93.46

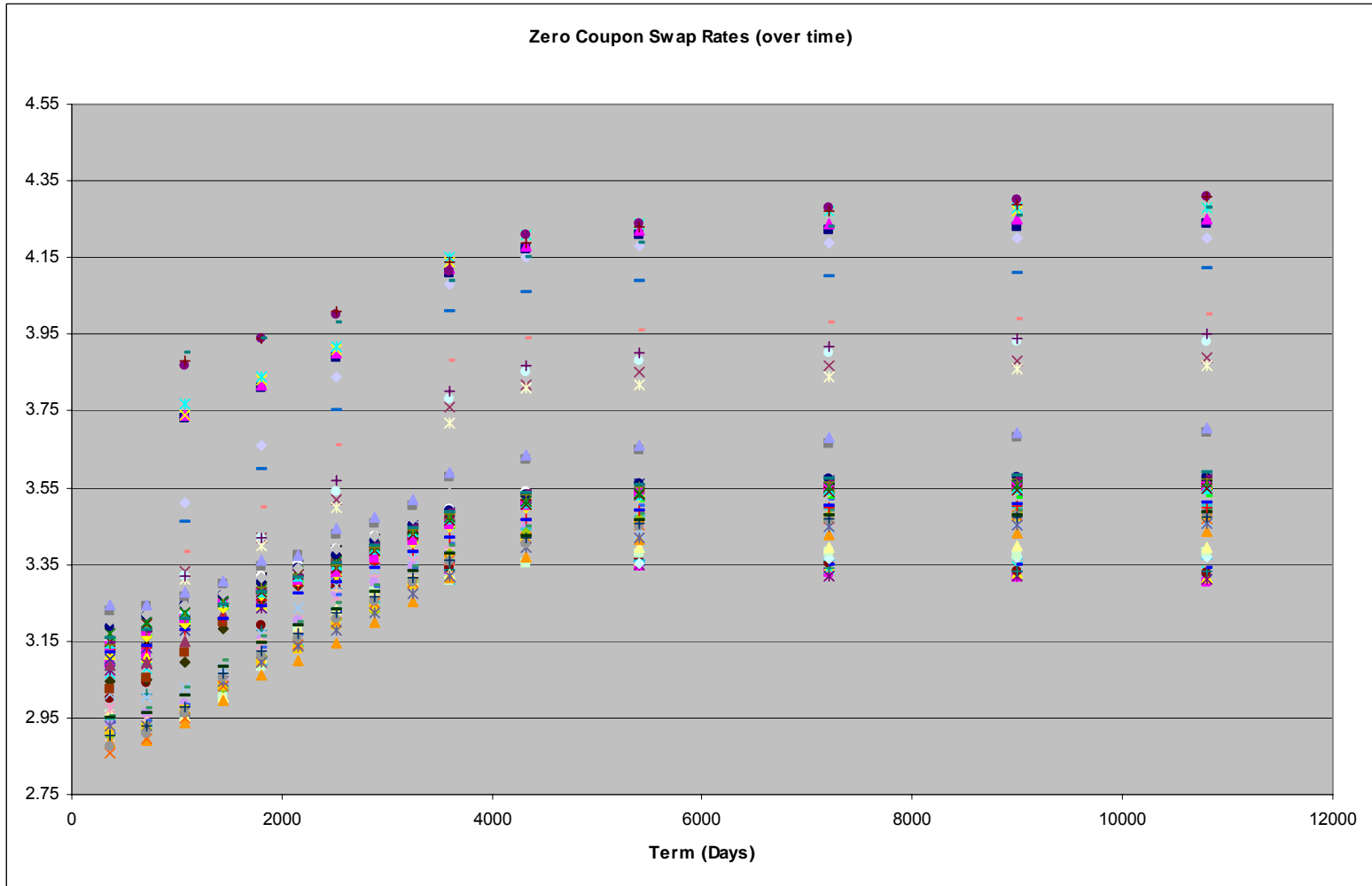
AVERAGE:
  3YR      5YR      7YR      10YR      12YR      15YR      20YR      25YR      30YR
3.85/95  3.89/99  3.93/03  4.04/14  4.10/20  4.14/24  4.18/28  4.21/31  4.23/33

ELIGIBLE CONTRIBUTORS RATES
  3YR      5YR      7YR      10YR      12YR      15YR      20YR      25YR      30YR
3.80/90  3.89/99  3.92/02  4.02/12  4.07/17  4.12/22  4.16/26  4.18/28  4.21/31
3.85/95  3.90/00  3.95/05  4.04/14  4.10/20  4.14/24  4.18/28  4.21/31  4.24/34
3.86/96  3.89/99  3.95/05  4.04/14  4.09/19  4.15/25  4.20/30  4.22/32  4.25/35
3.84/94  3.88/98  3.92/02  4.07/17  4.14/24  4.18/28  4.22/32  4.26/36  4.30/40
3.87/97  3.88/98  3.93/03  4.05/15  4.10/20  4.15/25  4.20/30  4.20/30  4.21/31
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3.83/93  3.87/97  3.89/99  4.07/17  4.12/22  4.14/24  4.19/29  4.20/30  4.21/31
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3.85/95  3.90/00  3.93/03  4.03/13  4.09/19  4.13/23  4.17/27  4.20/30  4.21/31

BANKS CONTRIBUTING TODAY:ABN  CBA  CITI  DB  GS  JP  MBL  NAB  RBS  UBS  LEH
    
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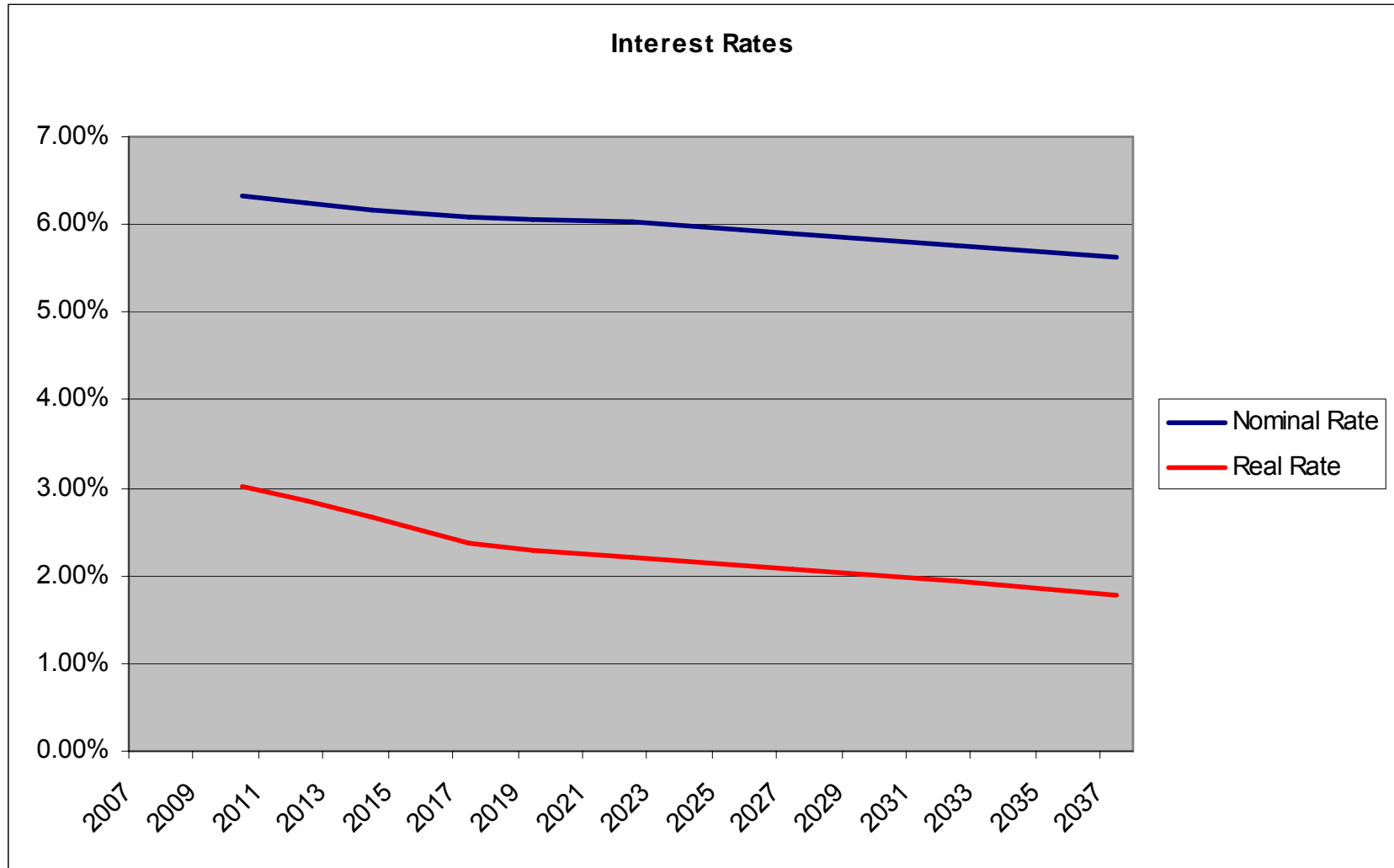
**Solved problem of obtaining better estimates for longer dated inflation
but weekly data available only**

Linker Swap Market



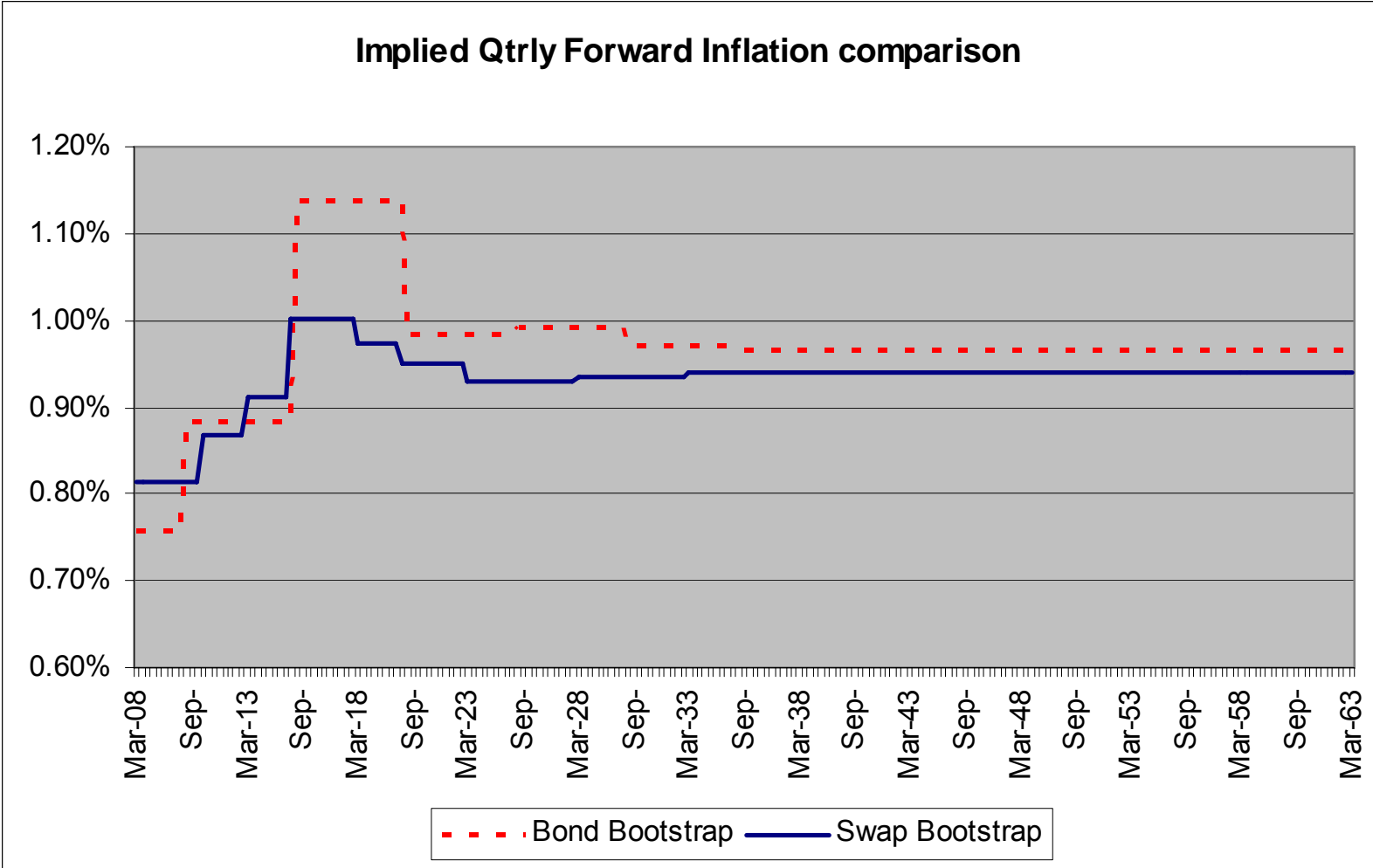
The direction of the zero swap rates over 2008

Linker Swap Market



Real Yield curve implied from the zero coupon swap market

Which is the true inflation estimate?



Model Axiom – hedging should be consistent with the chosen bootstrap

Simple Hedging Recommendations

Directional Derivatives

Definition: The partial derivatives of f at x_0 are the directional derivatives in the directions $\{e_1, \dots, e_n\}$

If f and all of its partial derivatives up to order k exist and are continuous then f is said to be in $C^k(D)$ where D is an open set in R^n

If f is in $C^k(D)$ then $\frac{d}{dx_1} \left(\frac{df}{dx_2} \right) = \frac{d}{dx_2} \left(\frac{df}{dx_1} \right)$

Applied to hedging:

Let $\tilde{y} = (y_1, \dots, y_n)$ be a collection of real yields, usually a bootstrapped curve, and let $V(\tilde{y})$ be the valuation of an inflation-linked transaction then

$$DV(\tilde{y}) = \left(\frac{dV(\tilde{y})}{dy_1}, \frac{dV(\tilde{y})}{dy_2}, \dots, \frac{dV(\tilde{y})}{dy_n} \right)$$

If $B(y_i)$ is the valuation of a real security at some yield y_i and the derivative $\frac{dB(y_i)}{dy_i}$

exists then the standard directional hedging recommendation is given by the ratio

$$\frac{dV(\tilde{y})}{dy_i} \cdot \frac{dB(y_i)}{dy_i}$$

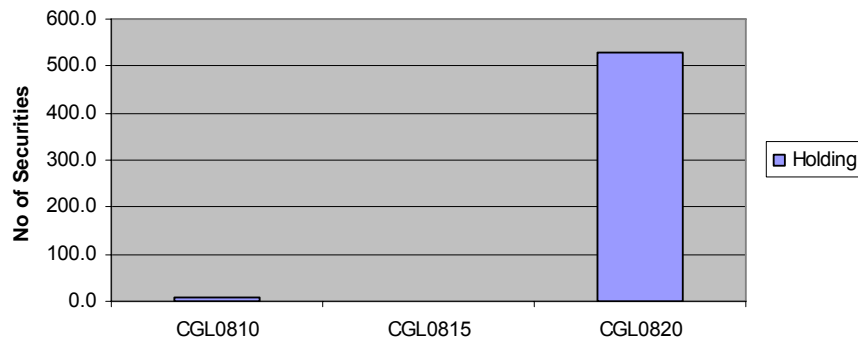
Comparing the two markets - Hedging Recommendations

Example:

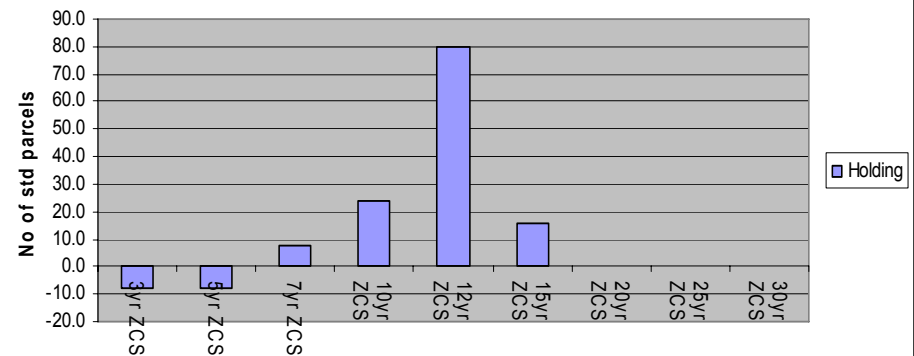
13yr CIB-type Coupon swap test trade

Sensitivity to a 1bp movement in real yields

CGL Hedge Portfolio (Example)



ZCS Hedge Portfolio (Example)



Model Axiom – hedging should be consistent with the chosen bootstrap

Comparing the two markets

Inflation Bonds

Implied Inflation

- **Extrapolation required**
- **Potential credit pollution**
- **Problems with supply/demand**

Hedging

- **Heavy emphasis on the CGL0820 security**
- **Potential distortions due to demand**
- **Must be sourced from the sensitivity behaviour of the bond bootstrap**

Main Point

- **Existing Market**
- **Widely used by many sectors in the economy – not just finance sector**
- **Little innovation**

Zero Coupon CPI Swaps

Implied Inflation

- **Longer Dated estimates available**
- **No information for Years 1 and 2**
- **Better coverage in the mid terms**

Hedging

- **Greater hedging flexibility**
- **Market is not very deep at the moment**
- **Must be sourced from the sensitivity behaviour of the Zero Coupon swap bootstrap**

Main Point

- **Emerging Market in Australia**
- **Derivatives markets should evolve more sophisticated products more quickly**

Seasonality – Introduction

- Many markets show a seasonal pattern to the inflation statistics
 - UK market is monthly and exhibits a strong monthly seasonal pattern
- However, most markets have transparent pricing only for multiples of years
 - RPI market quotes
 - 1,2,3,4,5,6,7,8,9,10,15,20,30,40,50
 - AUD Market quotes
 - 3,5,7,10,12,15,20,25,30
- Thus, each trading house has to make its own assessment of seasonality cycle
- This can make it worth “shopping” around for pricing discrepancies between market makers

Seasonality – Model implications

The CPI time series can be expressed in a decomposed form. This decomposition could be summarized in the general parametric form

$$X_t = f(T_t, C_t, S_t, \varepsilon_t) \quad \forall t = 0, \dots, T$$

where:

- T_t is the global trend, usually taken as a smooth function that represents the long term evolution. This can be either a deterministic or stochastic process.
- C_t maybe a, potentially, longer economic or credit cycle
- S_t may be the seasonal or a shorter cyclical effect, corresponding to yearly fluctuations.
- ε_t is the random or irregular effect not included in the previous ones.
- f is a given function, that may be separable.

Seasonality – Model implications

The forward CPI market model introduced by E. Benhamou and N. Belgrade , assumes that the forward CPI is an exponential martingale under a certain probability measure:

$$\frac{dCPI(t,T)}{CPI(t,T)} = \mu(t,T)dt + \sigma(t,T)dW_t, \quad 0 \leq t \leq T$$

where:

- $CPI(t, T)$ is the CPI index observed at time t and fixing at time T ,
- $\mu(., .)$ & $\sigma(., .)$ are deterministic real functions,
- $W_t; t \geq 0$ is standard Brownian motion.

There are two techniques for incorporating seasonality to such a model:

1. To incorporate the seasonality in the coefficient of the diffusion either in the drift or in the volatility.
2. To consider seasonality as an exogenous variable ie define the stochastic diffusion for the deseasonalised forward CPI. The seasonality could then be a, multiplicative or additive, correction to the continuous time model.

Seasonality – Model implications

- Many of the market inflation models utilise the second option

$$\frac{d\tilde{CPI}(t,T)}{\tilde{CPI}(t,T)} = \mu(t,T)dt + \sigma(t,T)dW_t ; 0 \leq t \leq T \text{ in continuous time}$$

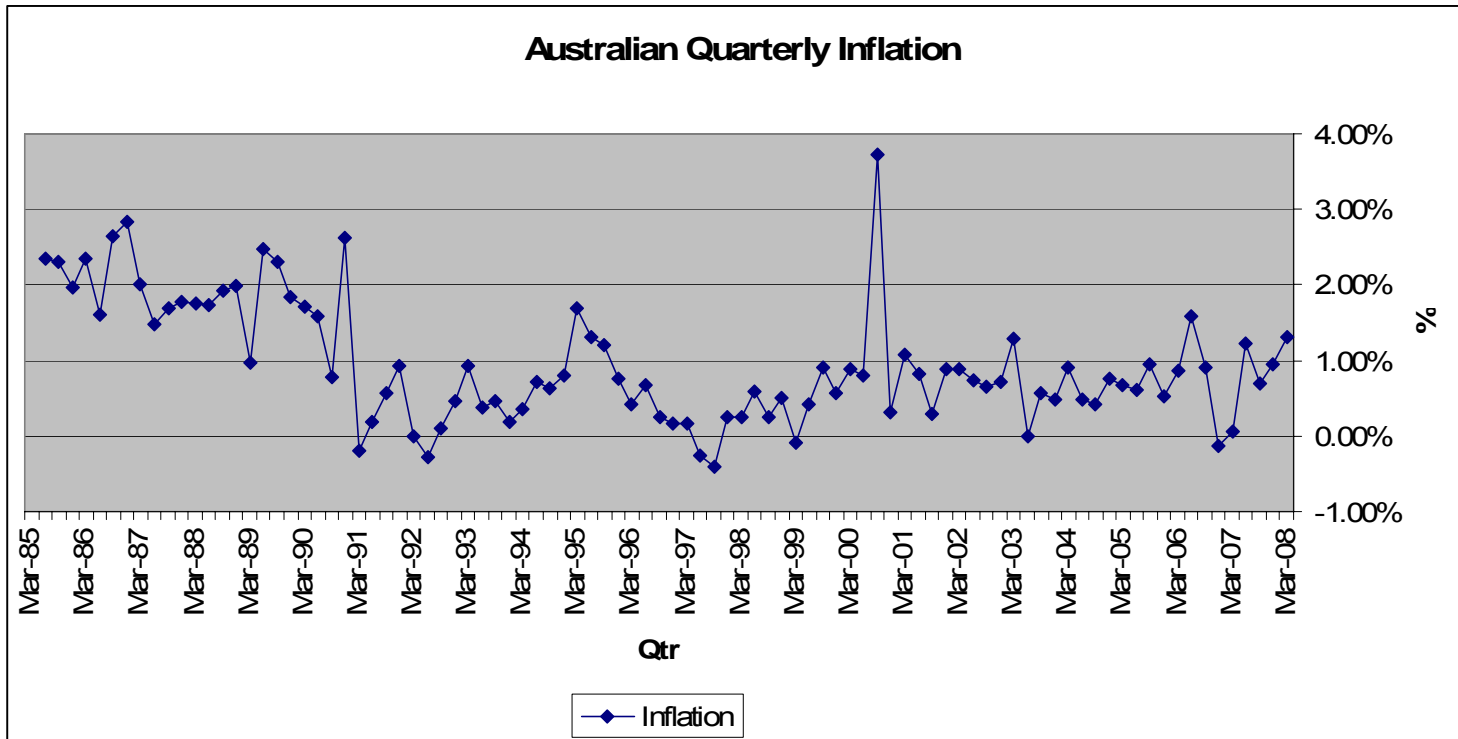
$$CPI(t,T) = \tilde{CPI}(t,T) + S_t ; \forall t = 0, \dots, T \text{ ie in discrete time}$$

- Fitting the stochastic process to the trend and long term cyclic components only
- Defining S to be a deterministic, periodical, sequence of period s such that the sum of all its elements equals zero i.e.

$$S_{t+s} = S_t ; \forall t = 0, \dots, T - s$$

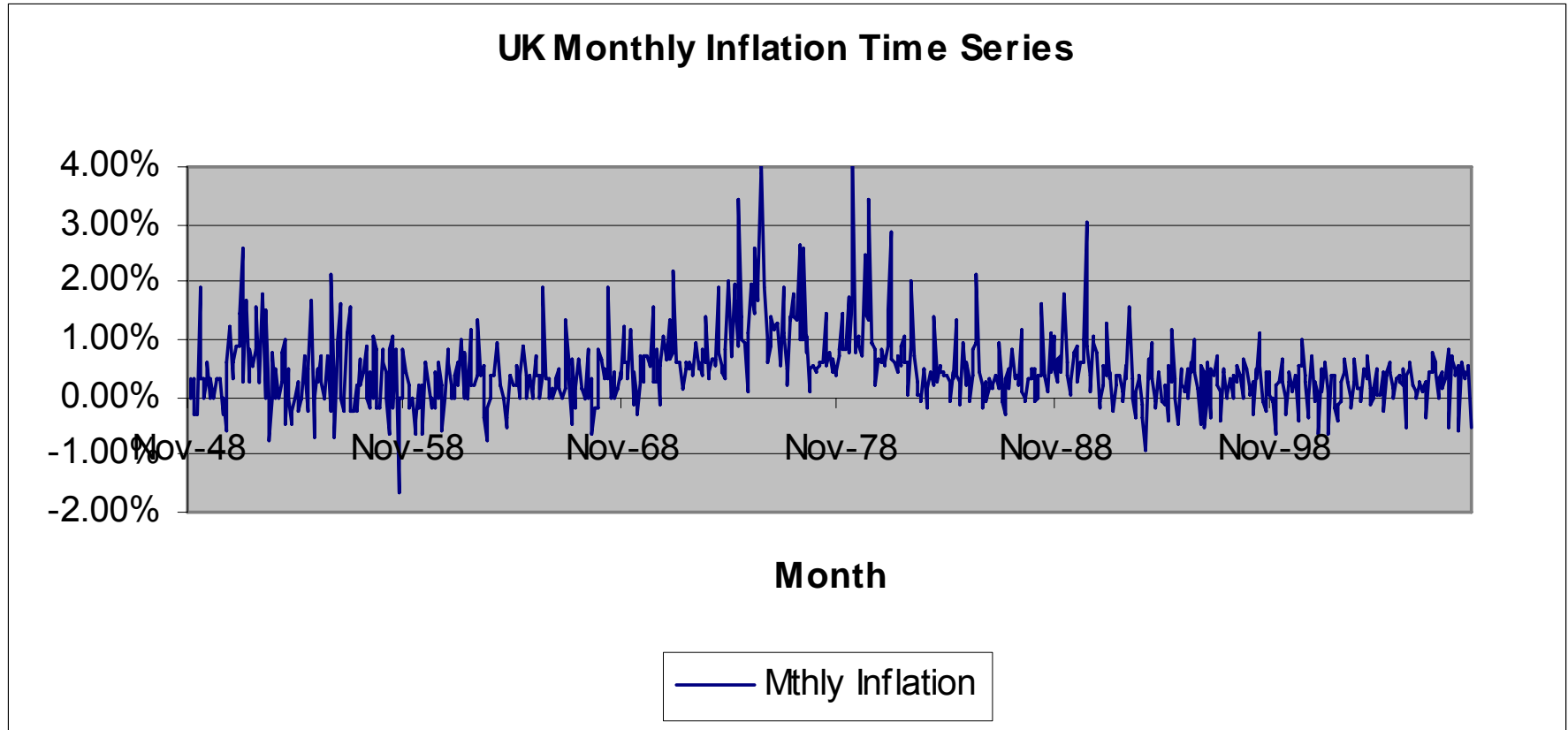
$$\sum_{t=1}^s S_t = 0$$

Seasonality in the Australian Market?



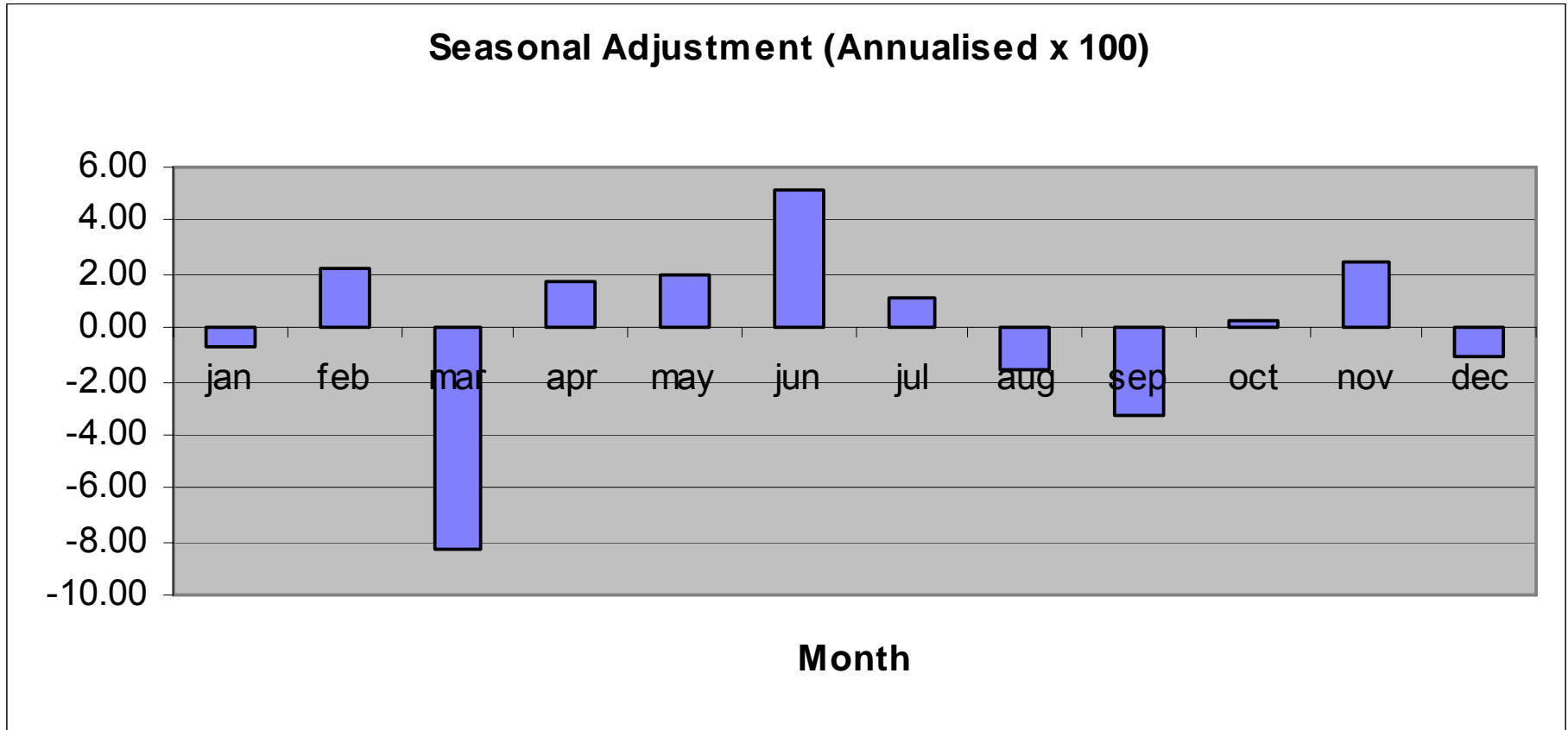
The Australian market exhibits no evidence of identifiable seasonality

Seasonality in the UK RPI Market



However, the UK Market does exhibit evidence of identifiable seasonality

Seasonality Estimates (UK RPI Market)



Seasonality can have a significant effect on the pricing of transactions that start mid year or have rolls that are not of length of one year

Seasonality - Hedging

So what does a seasonality sensitivity actually mean?

Let $\tilde{S} = (S_1, \dots, S_s)$ be the collection of seasonality adjustments and let $V(\tilde{S})$ be the valuation of an inflation-linked transaction then, as per usual,

$$DV(\tilde{S}) = \left(\frac{dV(\tilde{S})}{dS_1}, \frac{dV(\tilde{S})}{dS_2}, \dots, \frac{dV(\tilde{S})}{dS_s} \right)$$

.... but what would a hedging portfolio look like?

If you are a two way player in the market then your portfolio may, to some extent, be self hedged.

Otherwise...?

Without an inflation forward or an inflation futures market, changes in the seasonality pattern will be difficult to properly hedge.

Seasonality – Hedging assuming a regular cycle

If the seasonal cycle is really regular then a directional derivative $\frac{dV(\tilde{S})}{dS_j}$ can be defined:

$$\begin{aligned}\frac{dV(\tilde{S})}{dS_j} &= V(\hat{S}) - V(\tilde{S}) \\ &= V(S_1 - \varepsilon_1, \dots, S_j + \Delta s, S_i - \varepsilon_i, \dots, S_s - \varepsilon_s) - V(\tilde{S})\end{aligned}$$

where

$$\hat{S}_{t+s} = \hat{S}_t; \quad \forall t = 0, \dots, T - s$$

$$\hat{S}_j = S_j + \Delta s; \quad \hat{S}_i = S_i - \varepsilon_i; \quad i \neq j$$

$$\sum_{i=1}^s \varepsilon_i I\{i \neq j\} = -\Delta s$$

$$\sum_{t=1}^s \hat{S}_t = 0$$

The choice for ε_i is a little subjective

Seasonality – is it really regular?

- However, the seasonal cycle behaves more like an irregular cycle
 - with changes in both the amplitude and the phase of the cycle over time.
- Although the period of the seasonal variation is, of course, always known there is considerable variation in shape.
- So looking at the impact of a “basis point” movement is probably of questionable use
- Few, if any of these shapes are reminiscent of a simple sine curve but it is well known fact that any periodic function with period p may be approximated by a sum of sine curves with periods $p, p/2, p/3, \dots$

Seasonality – a simple spectral perspective

- A simple spectral representation might be:

$$f(t) = a_0 + \sum_{j=1}^{\infty} a_j \sin\left(\frac{2\pi jt}{p} + \theta_j\right) = a_0 + \sum_{j=1}^{\infty} a_j \sin(\theta_j + \omega_j t); \text{ where } \omega_j = \frac{2\pi j}{p}$$

- If the pattern is not changing dramatically with time
 - fit fourier terms of period p , and the harmonics of this period, and subtract from the series.
- Otherwise
 - More sophisticated techniques, like complex demodulation, need to be applied to filter out the seasonal effects.
 - US Census Bureau X12-ARIMA model
 - Utilises many such techniques

So, perhaps we should be looking at the impact of changes in the seasonality “shape”

Seasonality – a simple spectral perspective

- The X-12 Arima model returns the “average” seasonality of every “sub-series” of the N periods.
 - Consequently, it is possible to think of a “variance” around the average seasonal process
- A simpler method might be to just fit the ARIMA model over different time slices.
 - This should return different filters and consequently different seasonal cycles.
- However, this risk is difficult to properly hedge
 - one way of managing this risk would be by raising a model risk provision against the “worst case” seasonal cycle
 - as determined by the current trade book

Fisher Equation – an Overview

- Fisher Equation : $1+i=(1+r)(1+\pi)$
 - I = ex-ante nominal interest rate
 - R = ex-post real interest rate
 - Π = ex-post inflation expectation over the same period
- Fisher relationship depends upon the nominal rates and the expected inflation rate moving freely
 - Really can only test this in a deregulated market
 - Need to select your time series carefully
 - The Australian market has only been deregulated since 1979
- Evidence of the effect is extremely mixed.
- In Australia, the empirical results suggest that there is:
 - a long-run co-integrating relationship
 - less evidence of any significant short run effect
- Macri (2007) suggests $i = 2.80 + 1.32 \times \pi$
 - Times Series 1979 - 2005

Fisher Equation - implications

- The Fisher relationship that relates nominal interest rates and expected inflation forms part of the core of macroeconomic analysis.
 - Despite the theoretical importance of this relationship, empirically it has met with little success.
- It is clear that the Fisher relationship is a “weak” relationship
- This has implications for bootstrapping
 - Trying to apply the relationship on corporate bonds is fraught with problems
 - Has consequences if trying to extend the bond inflation bootstrap beyond the risk free securities strip
- This has implications for modelling
 - Many models use the Fisher relationship to tightly tie the three markets together
 - It allows the evolution of only two of the tied triplet and then determine the third using this relationship
 - This needs to be applied with some degree of caution

Fisher Equation – Model implementation example

Define the nominal interest rate world. Let:

- P_T be the T-forward nominal world measure with associated Brownian motion $W(t)$
- β_t be the nominal bank account
- $B(t, T)$ be the T-forward nominal zero coupon bond

Define the real interest rate world. Let:

- P_T^f be the T-forward real world measure with associated Brownian motion $W^f(t)$
- β_t^f be the real bank account
- $B^f(t, T)$ be the T-forward real zero coupon bond

Define the inflation processes. Let:

- $S(t)$ be the spot, or current, CPI, under the nominal measure P_T .
- $S_T(t)$ be the forward CPI process commencing at time, t , and finishing at time, T .

Fisher Equation – Model implementation example

The SDE for the real zero coupon Bond under the real world measure is:

$$\frac{dB^f(t,T)}{B^f(t,T)} = r^f(t)dt - \left(\int_t^T \sigma^{f*}(t,u)du \right) dW^f(t)$$

If one assumes that an amount invested in a real bank account and converted into nominal dollars at maturity is the equivalent to a nominal asset then the SDE for the real zero coupon bonds under the nominal measure is:

$$\frac{dB^f(t,T)}{B^f(t,T)} = r^f(t)dt - \left(\int_t^T \sigma^{f*}(t,u)du \right) (dW(t) - v(t)dt)$$

These equations show that the real world measure, P_T^f , can be related to the nominal world measure, P_T , by the measure change:

$$dW^f(t) = dW(t) - v(t)dt$$

$$\Rightarrow P_T^f = \in \left(\int_t^T v^*(s)dW(s) \right) P_T$$

where \in represents Doolean Exponential

The spot CPI volatility relates the two measures

Fisher Equation – Model implementation example

It is usually assumed that the accumulation index (CPI) usually increases and so must have a positive drift $\mu(t)$ under the nominal measure P

$$\frac{dS(t)}{S(t)} = \mu(t)dt + v^*(t)dW(t)$$

Again, asserting that an amount invested in a real bank account and converted into nominal dollars at maturity is the equivalent to a nominal asset and so its discounted value must be a P martingale

$$\frac{d\left(\frac{\beta_t^f S(t)}{\beta_t}\right)}{\left(\frac{\beta_t^f S(t)}{\beta_t}\right)} = \left(\mu(t) + \left(r^f(t) - r(t)\right)\right)dt + v^*(t)dW(t)$$

but this has been defined as a P martingale and hence the CPI drift is "fixed" as the difference of the bond drifts $\mu(t) = r(t) - r^f(t)$

The spot CPI drift is constrained by the two bond drifts

Fisher Equation – Model implementation example

If an amount invested in a real bank account and converted into nominal dollars at maturity is the equivalent to a nominal asset then the zero coupon bonds can be related by

$$B^f(t, T) = \frac{S_T(t)}{S(t)} B(t, T)$$

under the nominal world P measure.

Since, we have consistent measures, we can find the SDE for $S_T(t)$ by

$$\frac{dS_T(t)}{S_T(t)} = \frac{d\left(\frac{B^f(t, T)S(t)}{B(t, T)}\right)}{\left(\frac{B^f(t, T)S(t)}{B(t, T)}\right)} = \left(\int_t^T \sigma^*(t, u) du\right) v_T(t) dt + v_T(t) dW(t)$$

$$\text{where } v_T(t) = v(t) + \left(\int_t^T \sigma^*(t, u) du\right) - \left(\int_t^T \sigma^{f*}(t, u) du\right)$$

The forward CPI volatility is a function of the spot volatility and the difference between the two interest rate volatilities

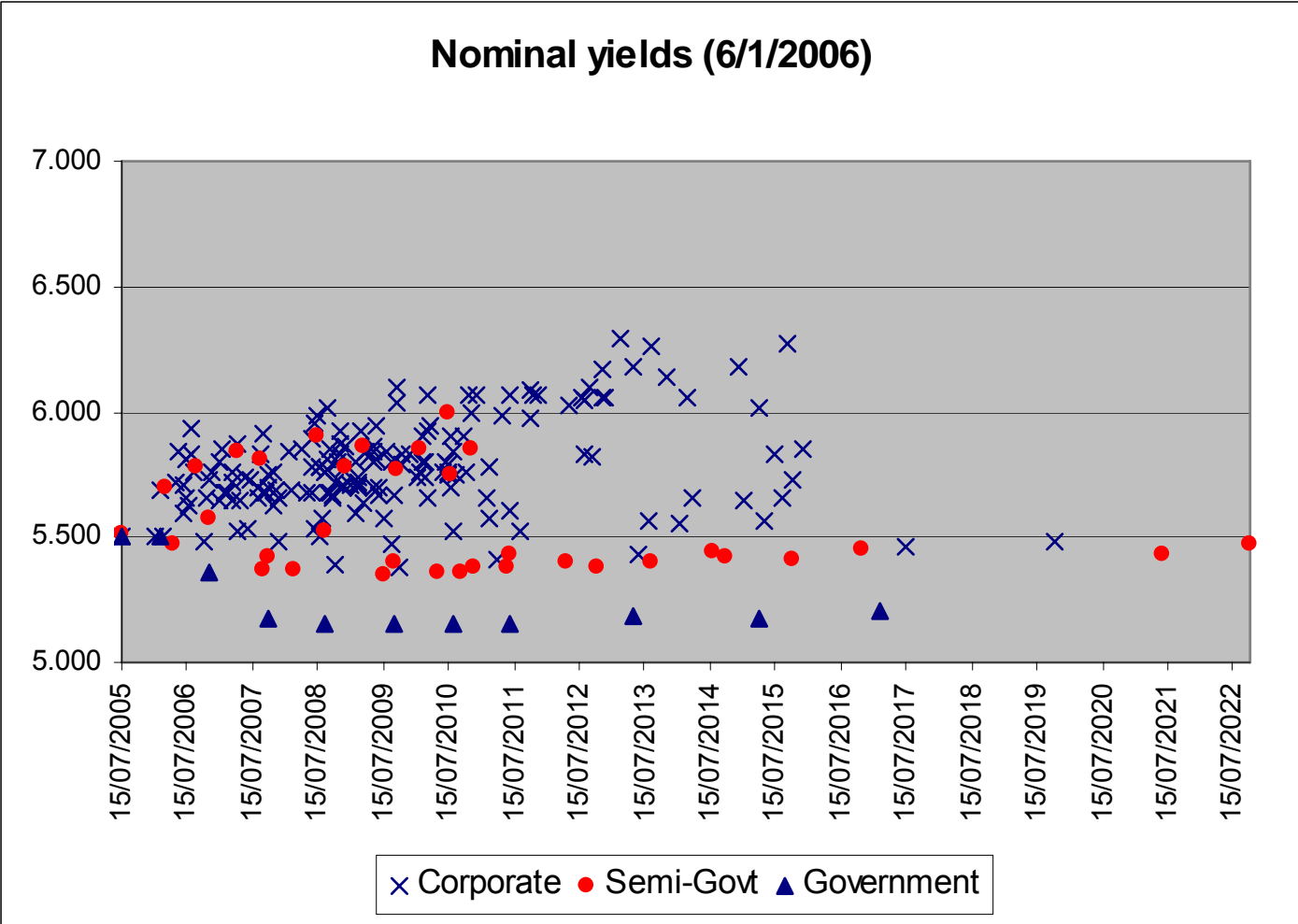
Fisher Equation – Model related issues

- The Fisher effect is meant to imply that market interest rates are good indicators of inflationary expectations.
- However, studies suggest that the Fisher hypothesis only holds weakly and hence it is not always true that CPI will drift in a consistent manner to the nominal and real bond drifts.
 - Unlike the FX market, there are, in Australia, few transparent instruments linking each market and hence it would be difficult to force realignment by arbitraging away any disparity.
- This has implications for modelling
 - Calibration is currently almost impossible
 - A transparent inflation options market does not exist in Australia
 - A transparent inflation forwards market does not exist in Australia
 - Perhaps the three processes should be allowed to evolve in a more loosely coupled manner

End of Section 1

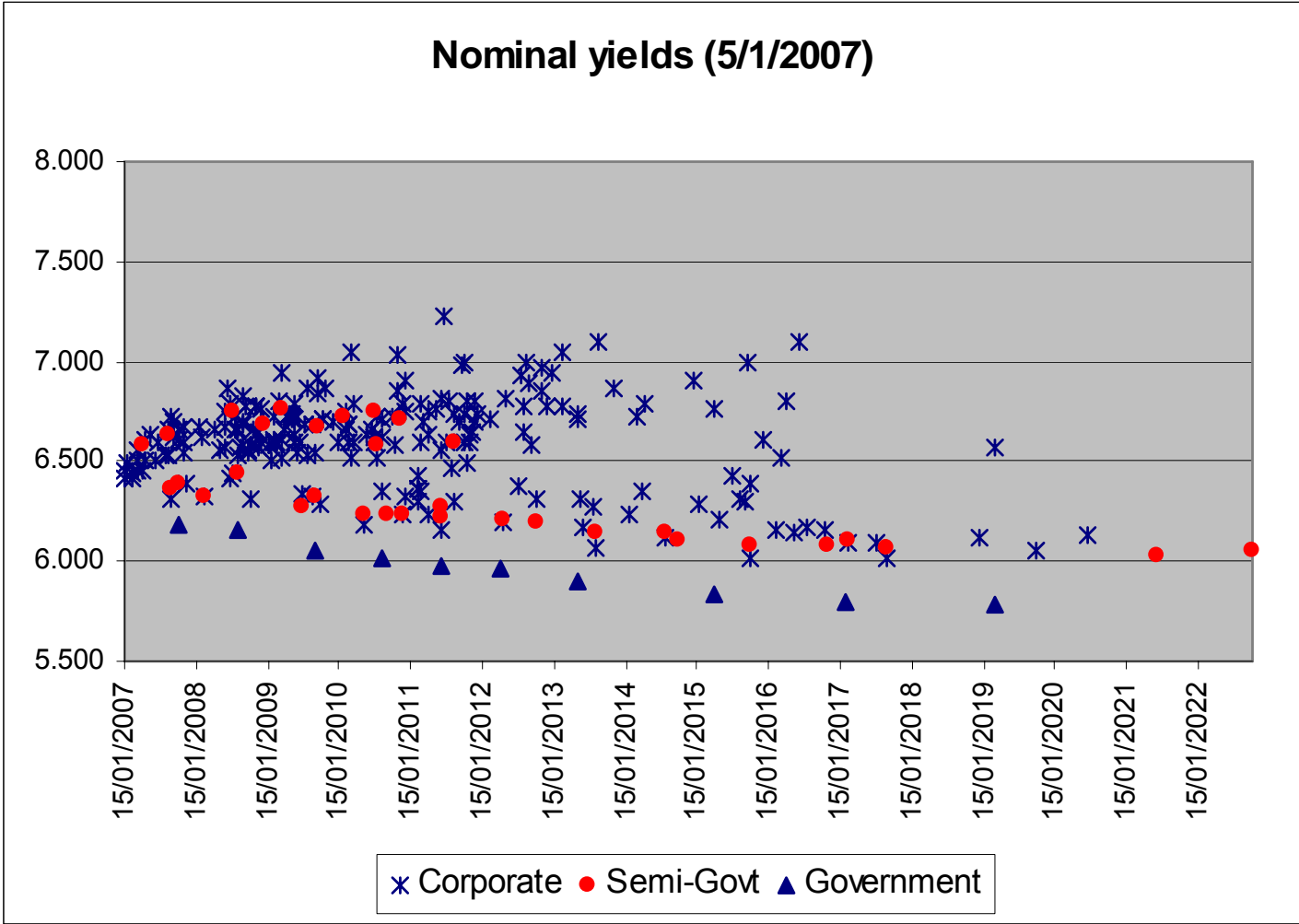
Section 2 – Credit Risk

Nominal Credit Spreads – Jan 2006



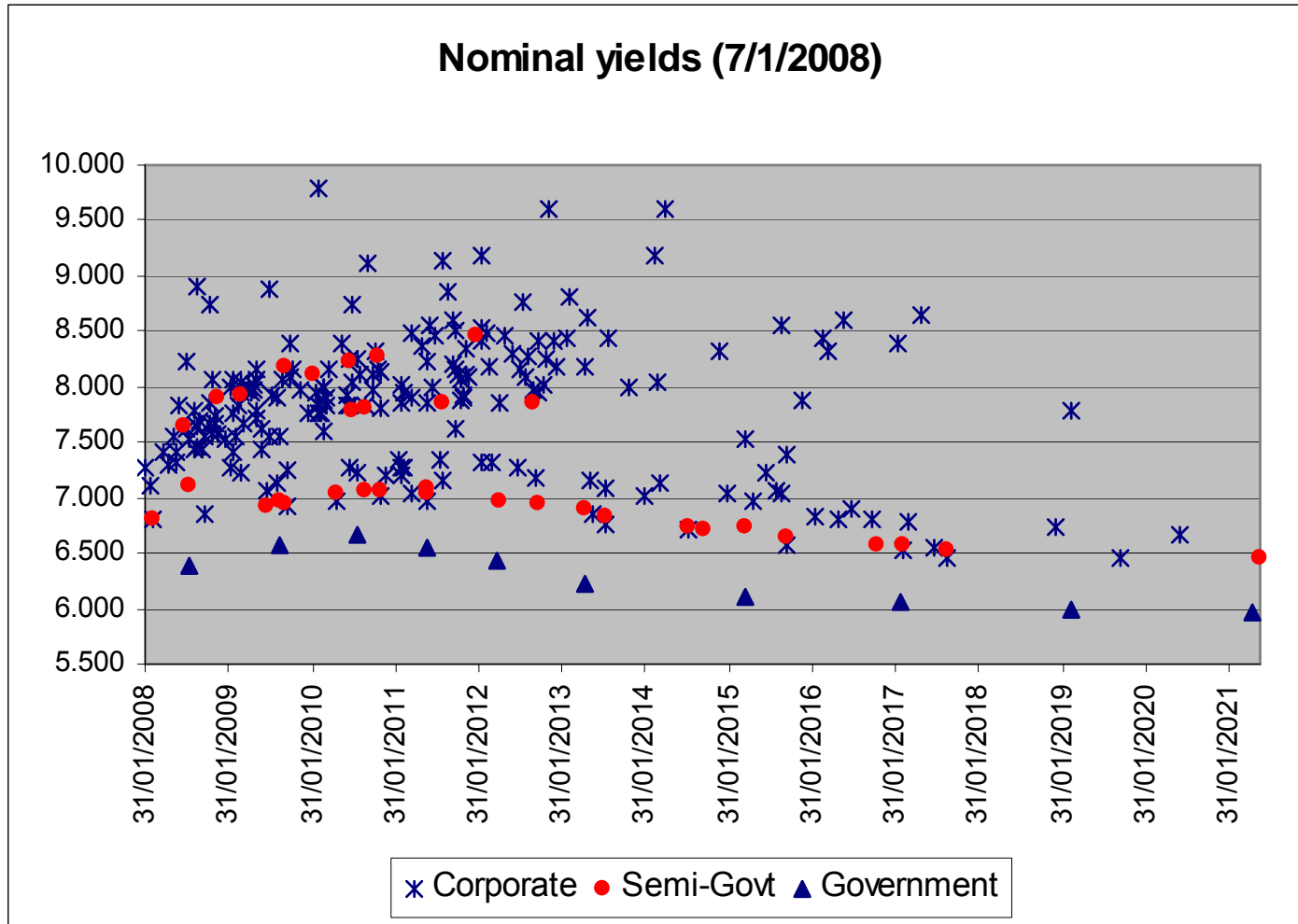
Nominal Credit Spread for Corp Bonds vs Risk free Bonds around 75bp

Nominal Credit Spreads – Jan 2007



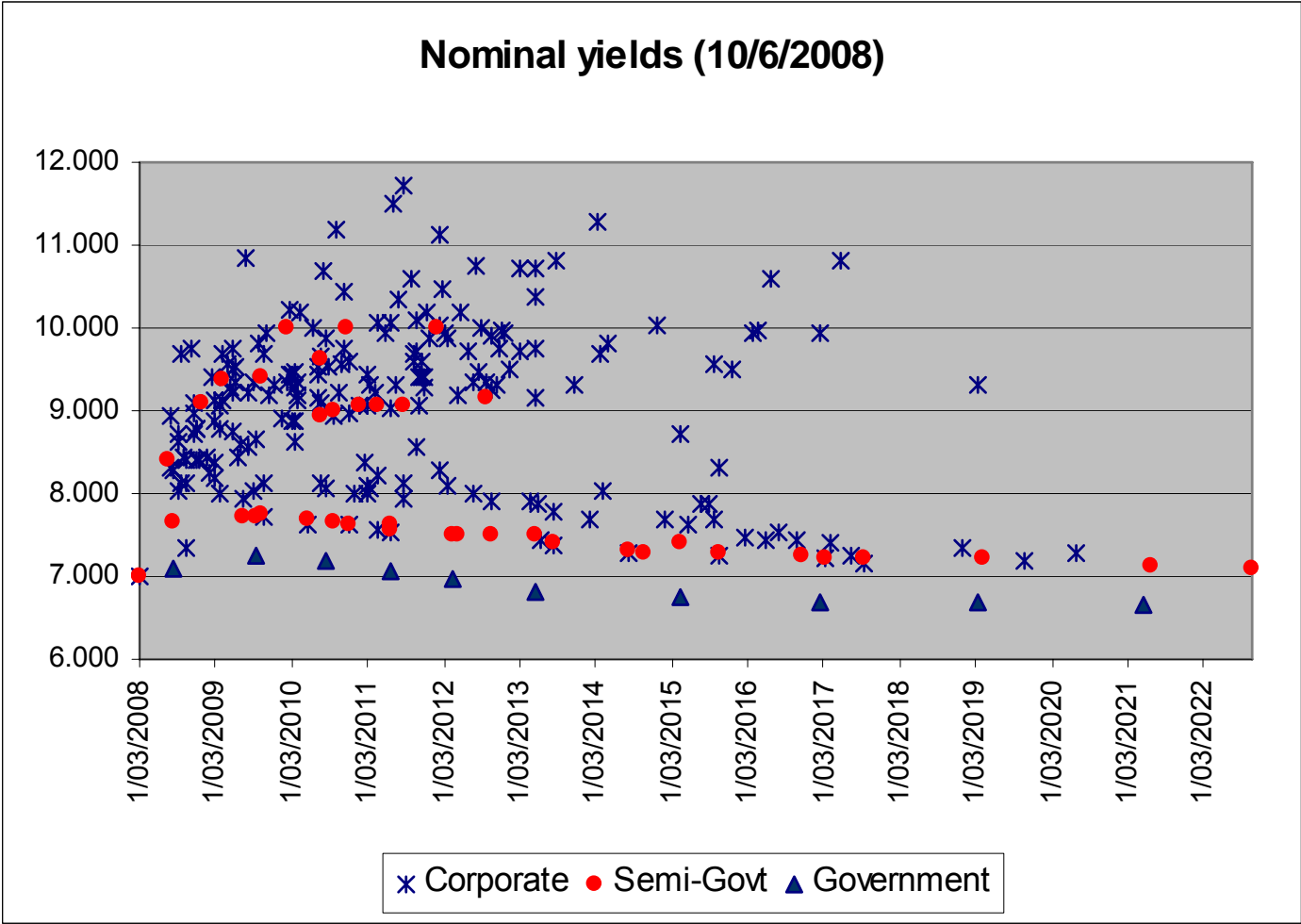
Nominal Credit Spread for Corp Bonds vs Risk free Bonds around 75bp

Nominal Credit Spreads – Jan 2008



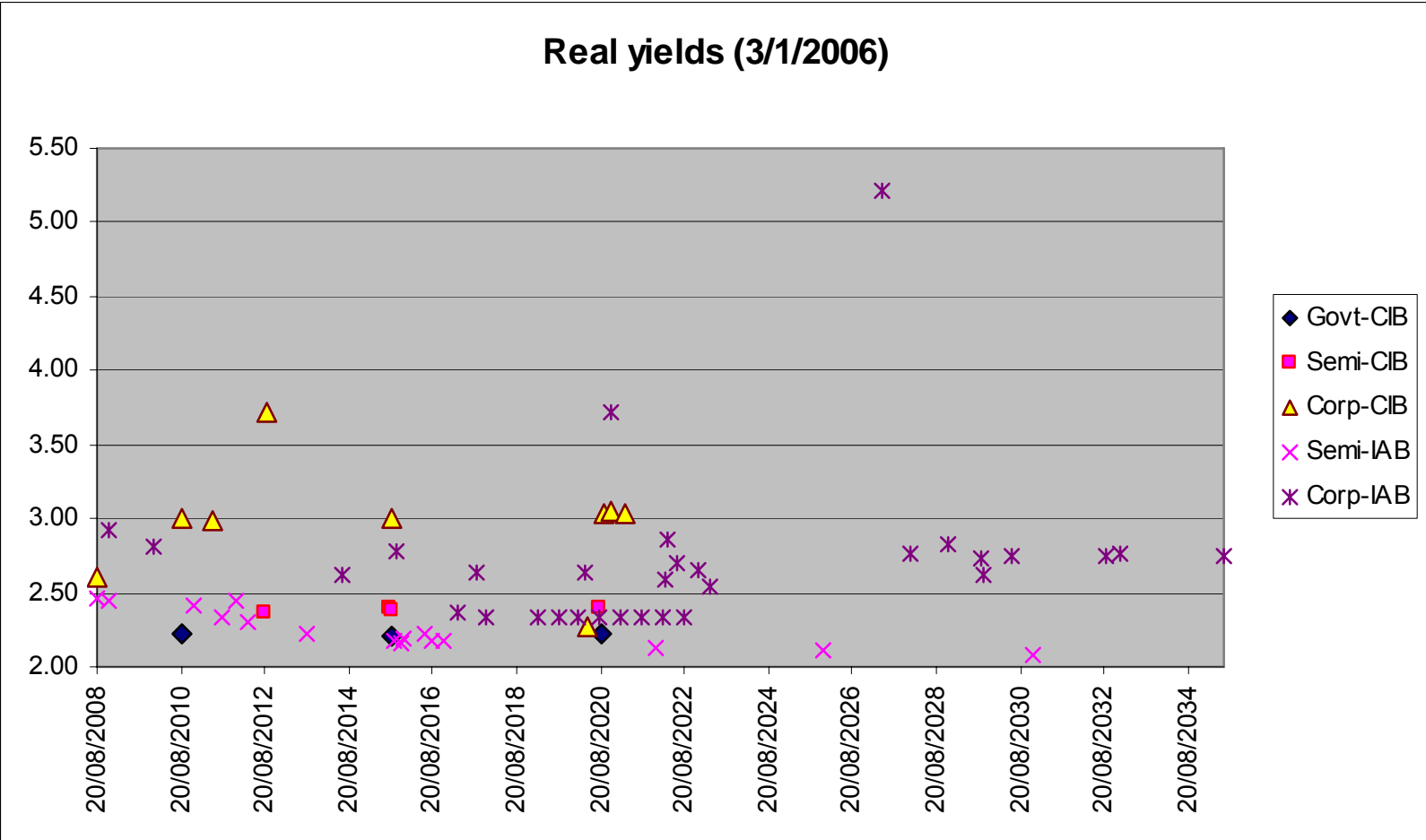
Nominal Credit Spread for Corp Bonds vs Risk free Bonds between 75bp-200bp

Nominal Credit Spreads – Jun 2008



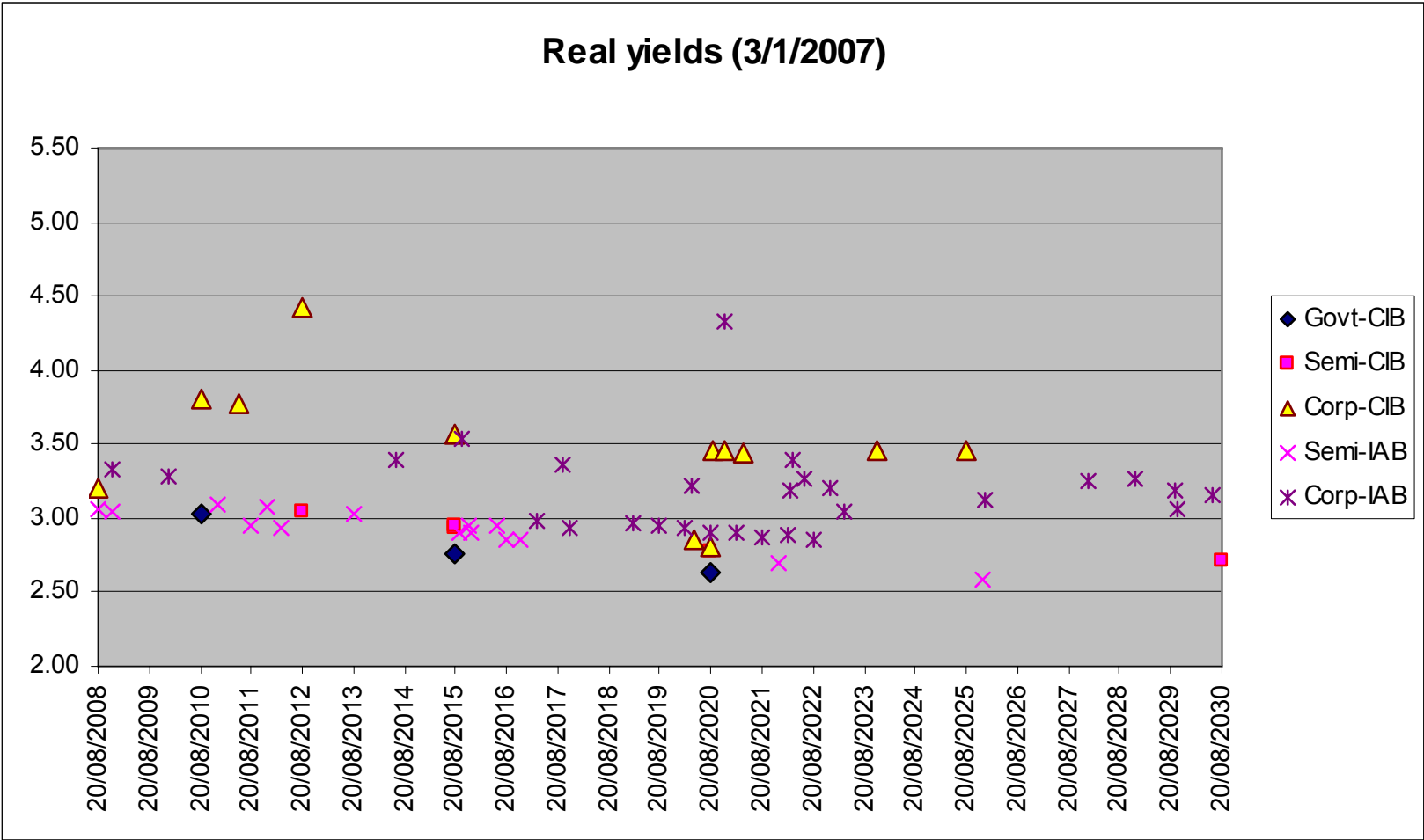
Nominal Credit Spread for Corp Bonds vs Risk free Bonds between 100bp – 250bp

Real Credit Spreads – Jan 2006



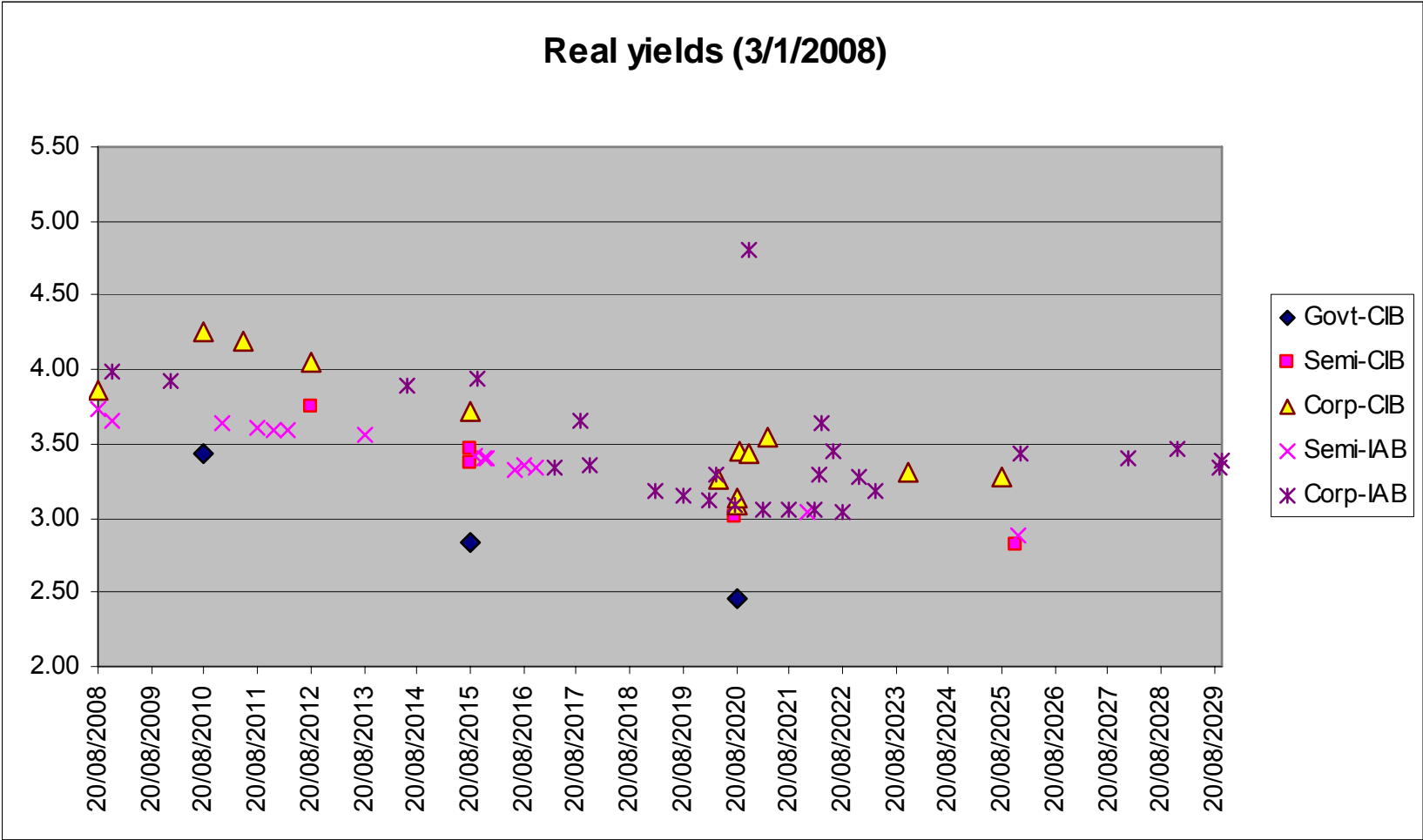
Real Credit Spread for Corp CIB vs Risk free CIB around 100bp

Real Credit Spreads – Jan 2007



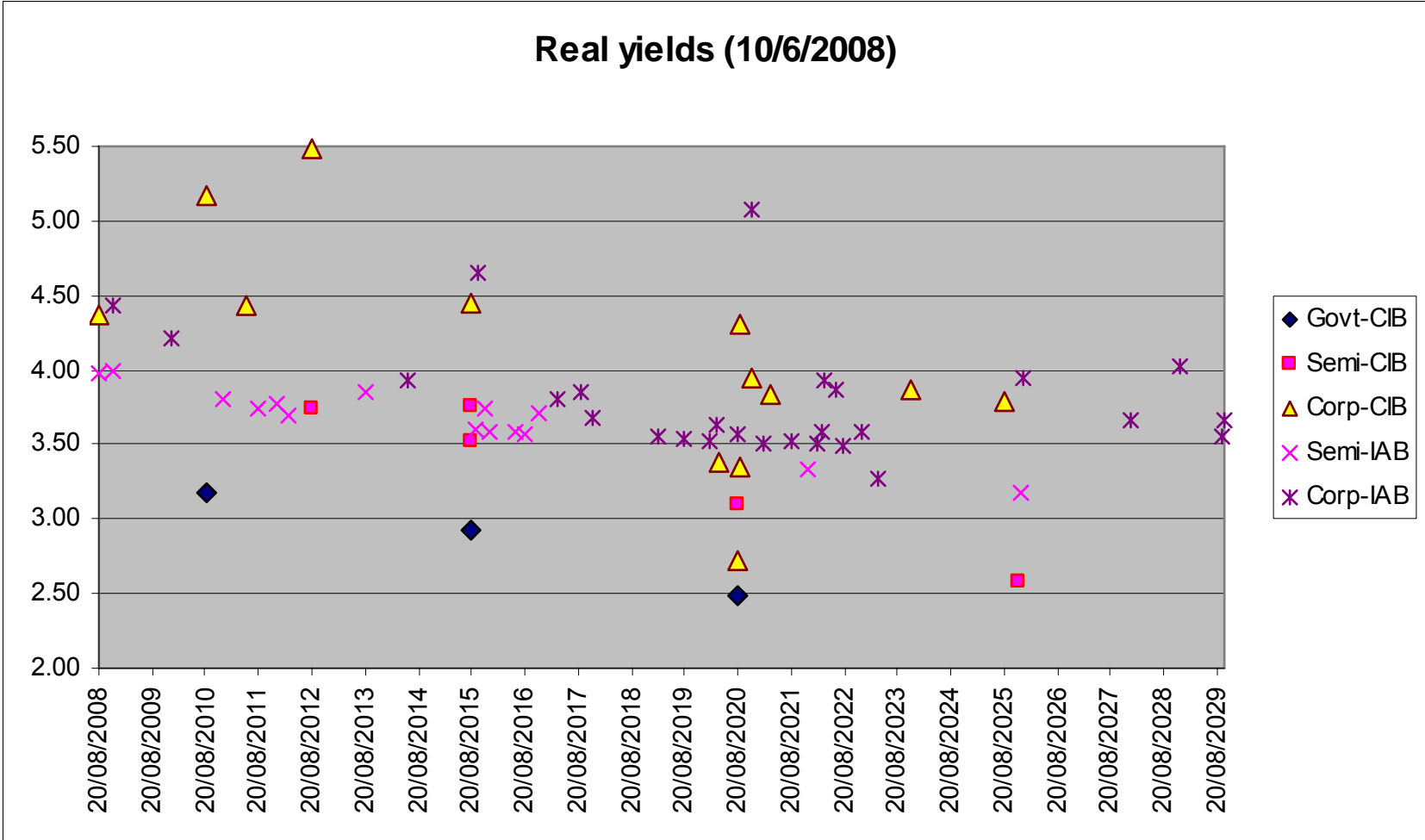
Real Credit Spread for Corp CIB vs Risk free CIB around 75bp-100bp

Real Credit Spreads – Jan 2008



Real Credit Spread for Corp CIB vs Risk free CIB around 100bp

Real Credit Spreads – Jun 2008



Real Credit Spread for Corp CIB vs Risk free CIB around 150bp-200bp

Credit Spreads - implications

- Real Bond credit spreads seem to be very sticky
 - Changes in real spreads lag changes in nominal spreads
 - Credit does not seem to be priced as effectively in the real markets
 - Primarily a price discovery problem – the market is basically a buy and hold market
 - Data quality introduces significant noise into the implied inflation estimates for credit risk instruments
- Further reinforces the view that the Fisher relationship is a “weak” relationship
 - Dangerous to apply it universally without understanding the underlying data
 - Trying to apply the relationship on corporate bonds is fraught with problems
 - Has consequences if trying to extend the bond inflation bootstrap beyond the risk free securities strip

Potential Credit Exposure

- Credit Risk
 - Risk of losing money if the counterparty defaults
 - Concern is when we are in the money
 - Potential Exposure estimated at 95% or 99% probability
 - Calculated at a counterparty level
 - Netting and Collateral mitigants
 - Basic calculation ignores recovery, probability of default, work-out costs, ...
 - Receiving Inflation/Paying Fixed
 - Win if Inflation higher than expected trajectory
 - Paying Inflation/Receiving Fixed
 - Win if inflation lower than expected trajectory
- Different methods are available to model this
 - % of Face Value estimates
 - Closed form approximations
 - Monte Carlo

The following slides consider raw exposure amounts only

Potential Credit Exposure – Monte Carlo

- The major issue : how do you project Inflation over time
 - project period on period inflation
 - Inflation can go negative
 - Underlying Gaussian model
 - project changes in CPI/Reference Index
 - CPI can drop but most people assume that this cannot happen
 - Underlying Log-normal model
 - project zero-coupon Swap Rates
 - Swap Rates can potentially be negative
 - Most pricing models assume an underlying Log-Normal Model
 - A Gaussian model is perhaps better for credit risk purposes

The key is to have a model that gives a very rich suite of curve types

Curve Evolution – Different Model alternatives

- HJM/BGM Cross Economy model
- Jarrow – Yildirim (2003)
 - Log-normal diffusion process for changes in the CPI/Reference Index
- Fabio Mercurio (2004, 2006)
 - Similar structure to the Jarrow-Yildirim model
- Belgrade – Benhounue - Koehler (2004)
 - Essential a multi-variate extension of the JY model
- Ornstein-Uhlenbuch
 - Normal diffusion process
- Nearly all of the models can only used in markets where
 - there is reasonably liquid inflation linked options market and
 - A reasonably transparent inflation futures/ forward market.
 - Neither of these trading markets currently exist in Australia

Most of the pricing models are not practical to apply in a Credit risk engine

A Simple Curve Evolution – Zero Coupon Swaps

Using the zero coupon swap market, we can define a multi-variate OU process

$$d\tilde{R} = \tilde{k}(\tilde{\theta} - \tilde{R})dt + \tilde{\sigma}^* dW(t)$$

where

$$\tilde{R} = \{R_i; i = (3y, 5y, 7y, 10y, 12y, 15y, 20y, 25y, 30y)\}$$

$$\tilde{\theta} = \{\theta_i; i = (3y, 5y, 7y, 10y, 12y, 15y, 20y, 25y, 30y)\} = \bar{\theta} \quad \forall i$$

$$\tilde{k} = \{k_i; i = (3y, 5y, 7y, 10y, 12y, 15y, 20y, 25y, 30y)\}$$

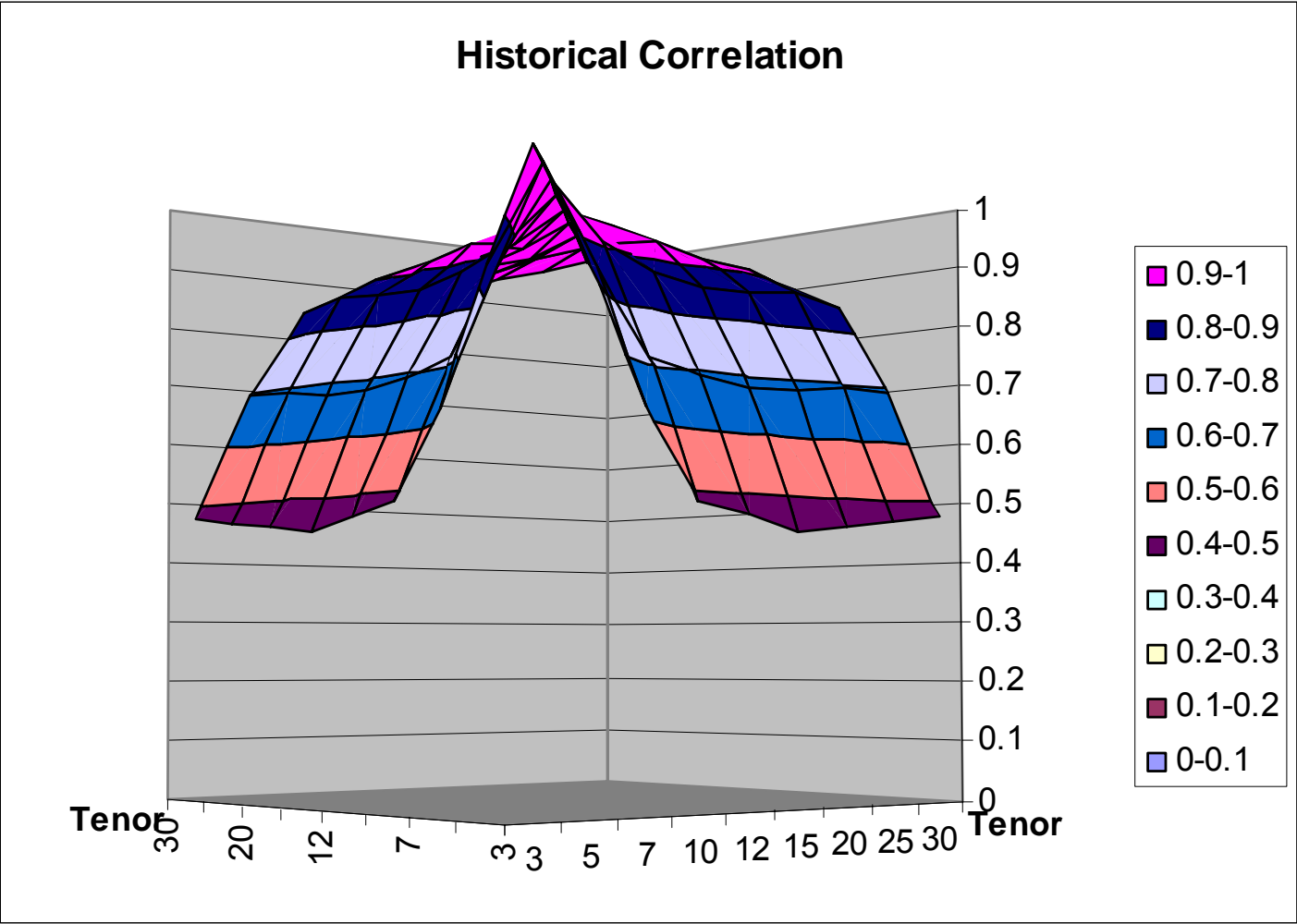
and using historical time series to estimate:

$$\tilde{\sigma} = \{\sigma_i; i = (3y, 5y, 7y, 10y, 12y, 15y, 20y, 25y, 30y)\}$$

$$\langle W_i(t), W_j(t) \rangle = \rho_{ij}$$

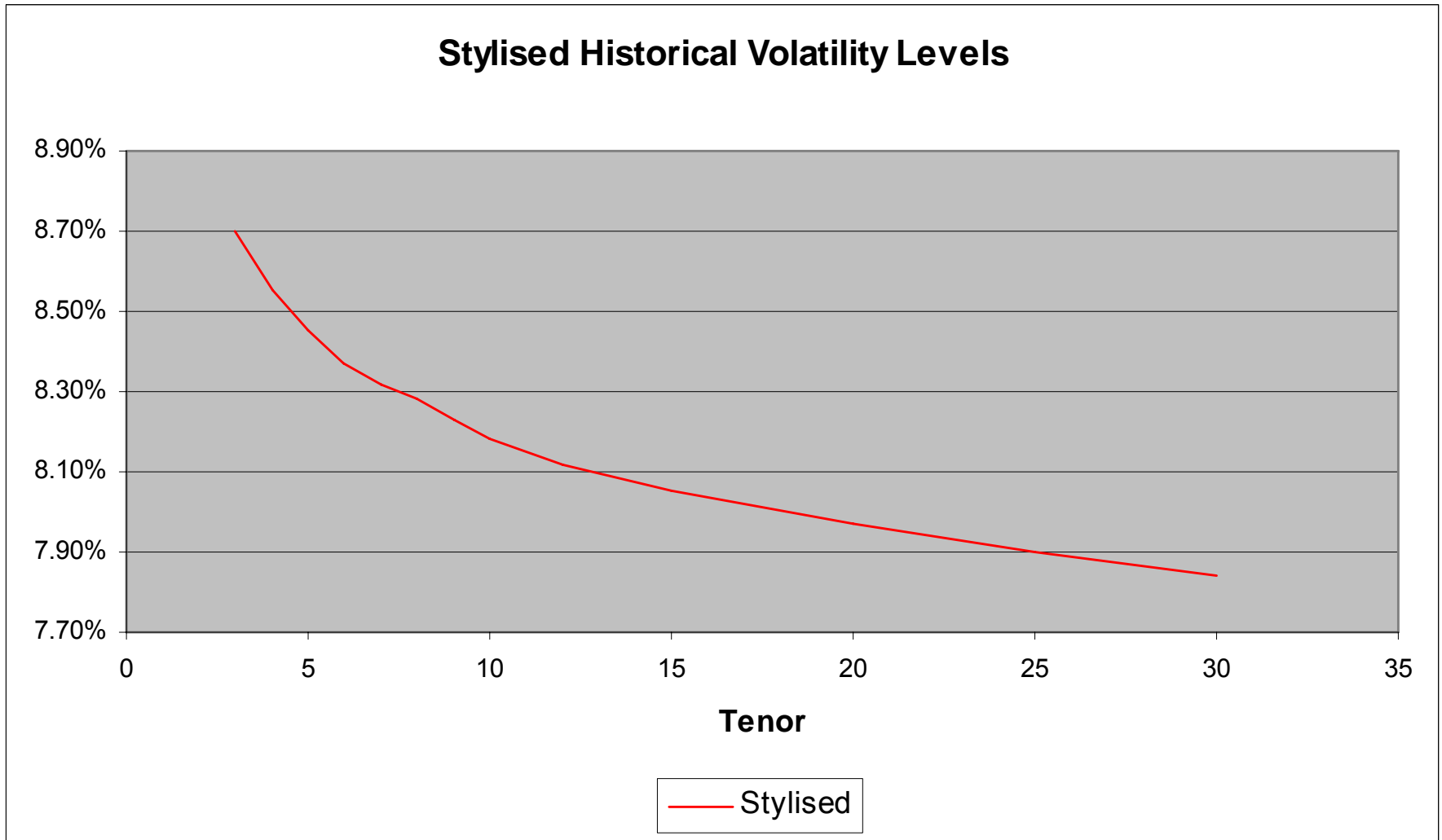
One short cut is to punt on the RBA and define Theta to be a function of the mid point of the official target inflation range

Curve Evolution – Historical Intra-ZCS spot Correlations



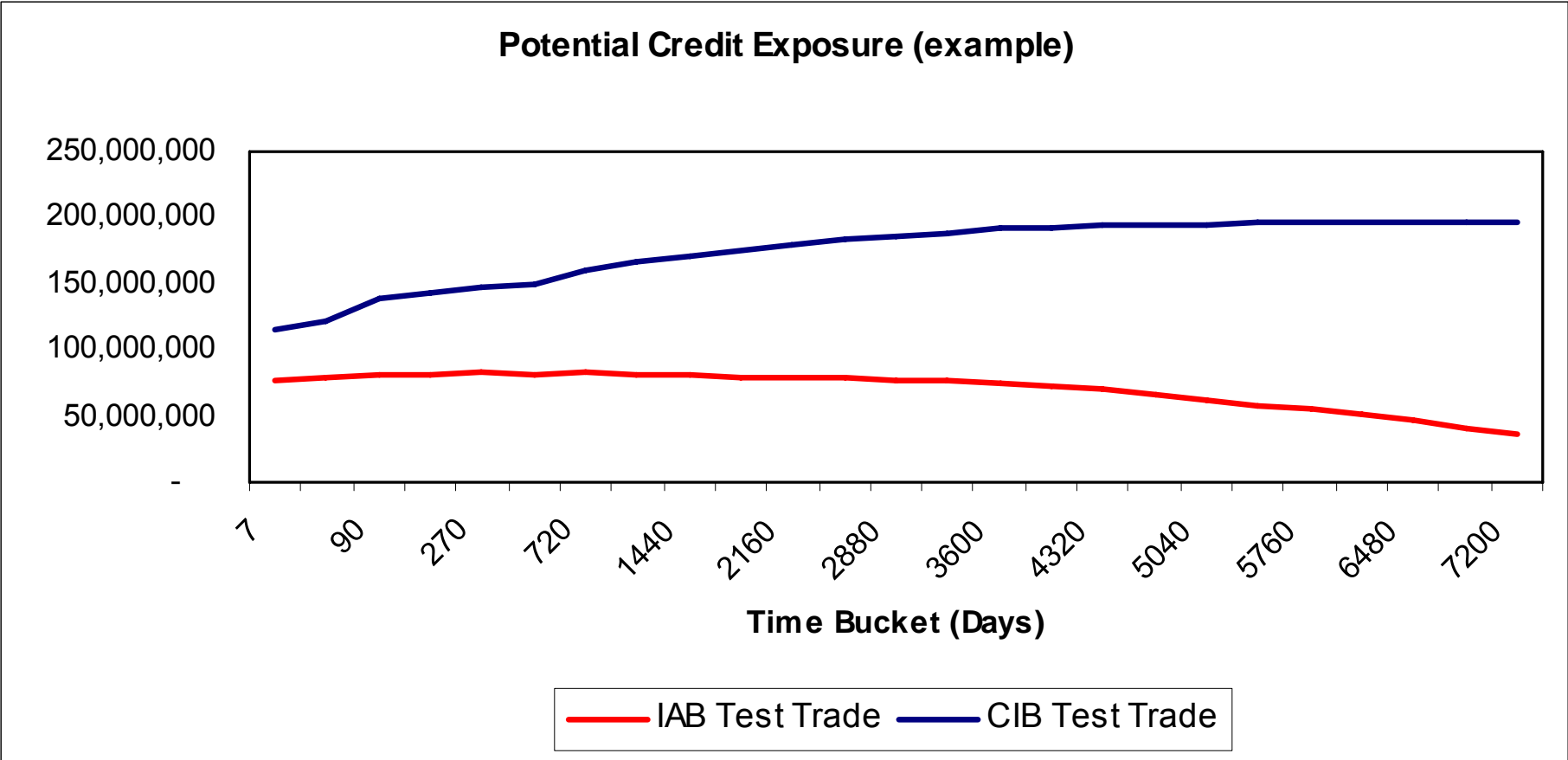
General Assumption: that correlation will decay exponentially

Curve Evolution – Historical ZCS spot Volatility



General Assumption: strong mean reversion at the back end of the curve

Potential Credit Exposure – Coupon Swaps



Remaining Cash flow profile and remaining Term to Maturity are the main drivers of the shape

Disclaimer

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