Risk-Based Solvency Capital Requirements for Insurance Undertakings

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Overview

- Solvency II and Swiss Solvency Test
- Pillar 1: Formal Setup
- Aggregation of Scenarios
- Group Effects and Diversification
- Example

Solvency II

- New EU-system to assess overall solvency based on prospective risk-oriented approach (initiated 2001)
- 3 pillars: quantitative requirements, supervisory activities, public disclosure
- Pillar 1: Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR), valuation of assets and liabilities, group ***diversification***, etc.
- Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) in charge to advice the Solvency II project through three specific calls for advice (Aug 2004-Mar 2006)
- Formal adoption of new framework directive by EC in July 2007

http://europa.eu.int/comm/internal_market/insurance/solvency_en.htm

Swiss Solvency Test (SST)

Regulator in Switzerland: Federal Office of Private Insurance (FOPI)

- Dec 2003: conceptual framework
- 2004: first version standard model
- 2004/05: field test: 10 big insurance companies
- 2005: field test: 45 insurers, 90% market share
- Jan 2006: SST becomes mandatory part of Insurance Supervisory Act
- 2006-2008: transition period for small companies, reinsurers and groups

http://www.bpv.admin.ch/themen/00506/index.html?lang=en

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Available Risk Capital (C)

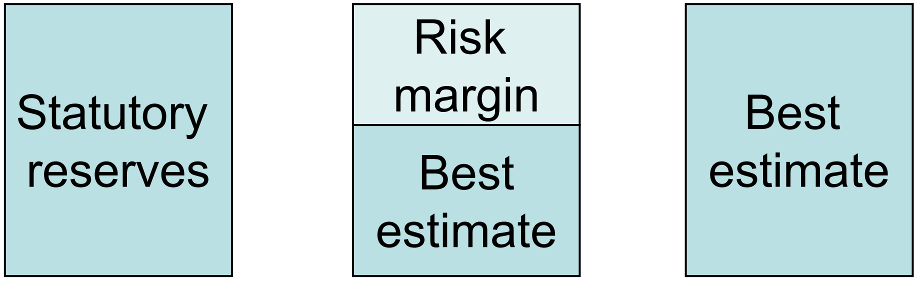
$$C = A - L$$

available capital — value of assets — value of liabilities

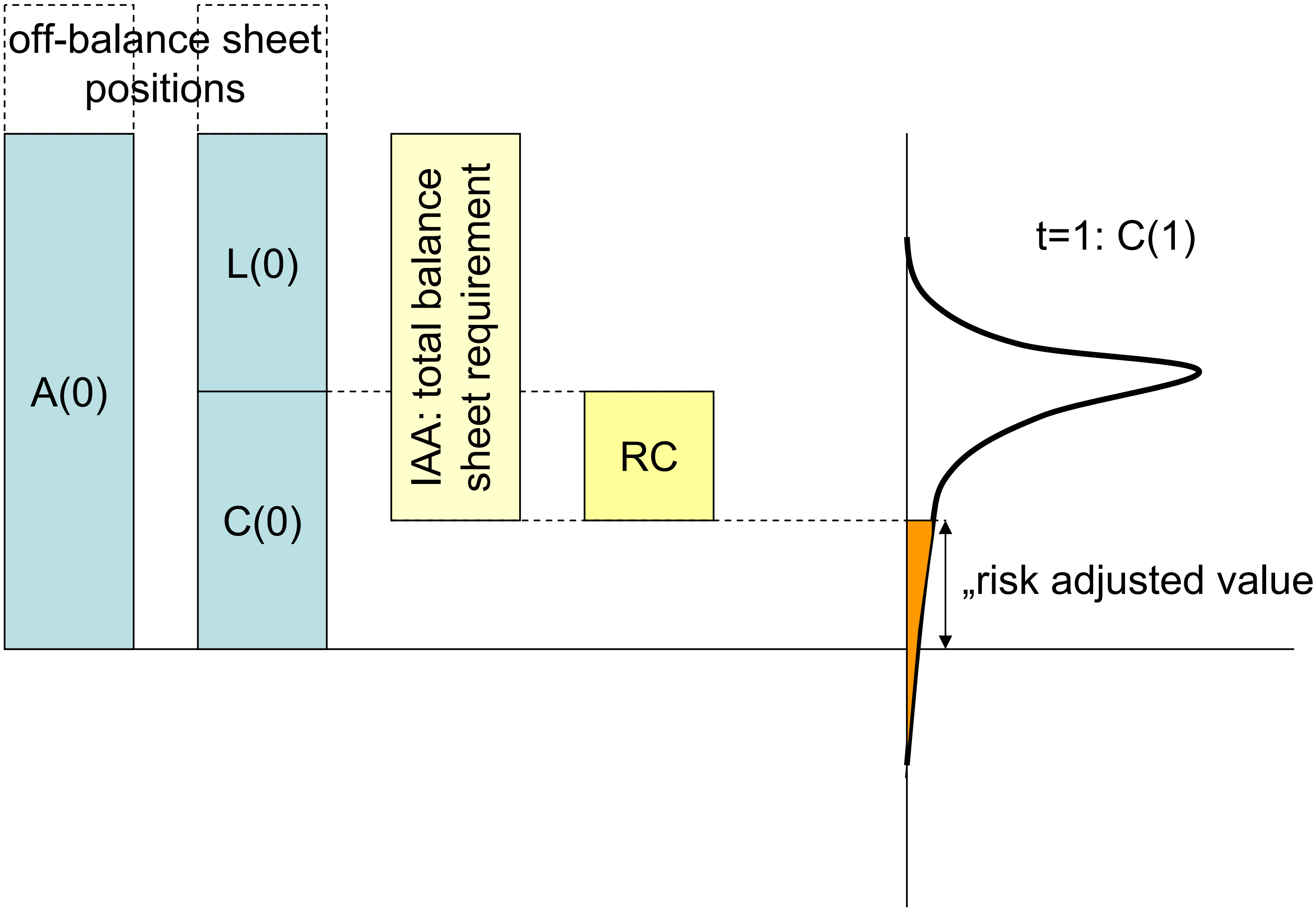
- Depends on choice of and valuation principles for assets and liabilities

- Market consistent valuation of assets:
    - marked to market if available
    - marked to model else (e.g. risk-neutral valuation)

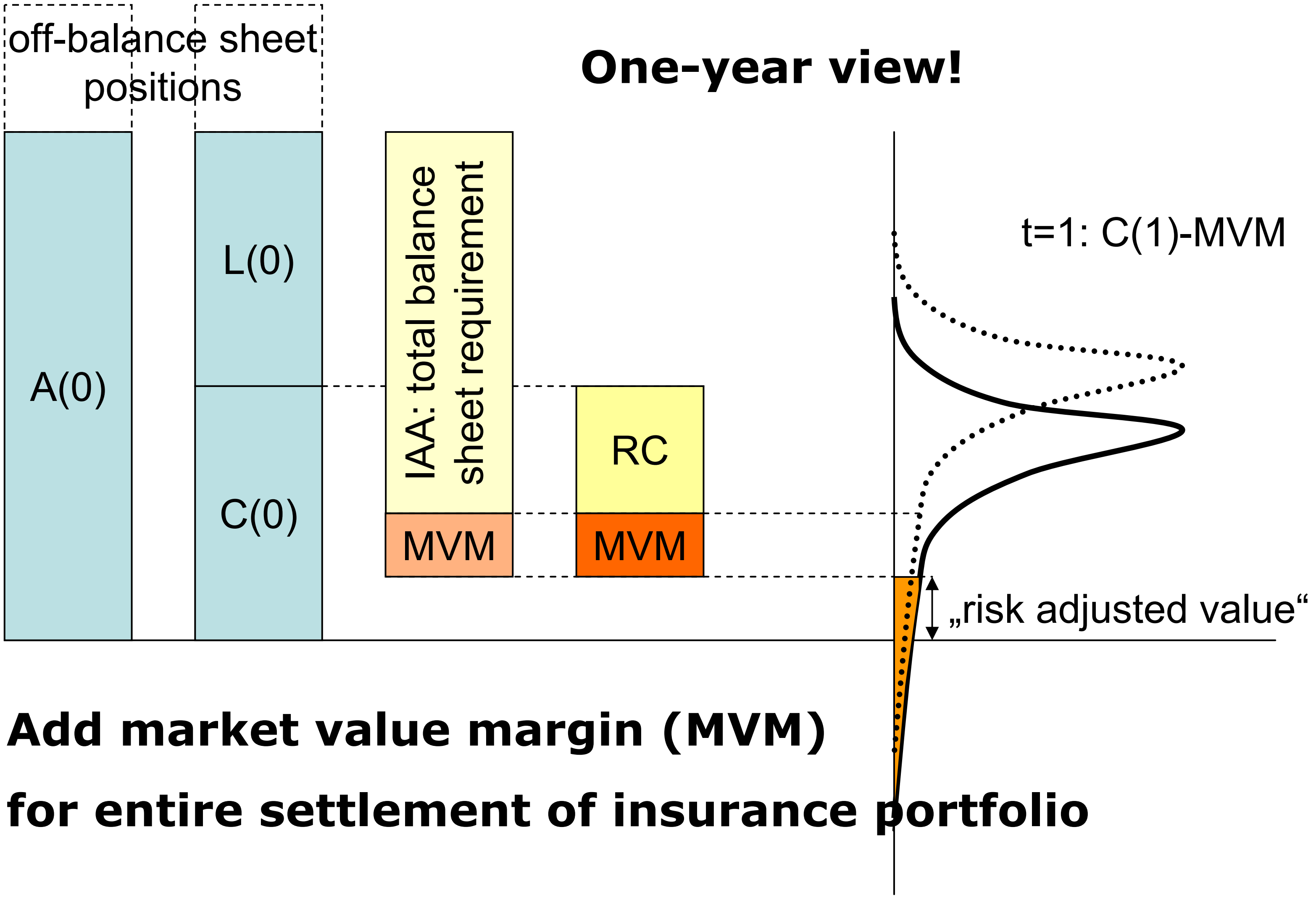
- Value of Liabilities:



Required Risk Capital (RC)



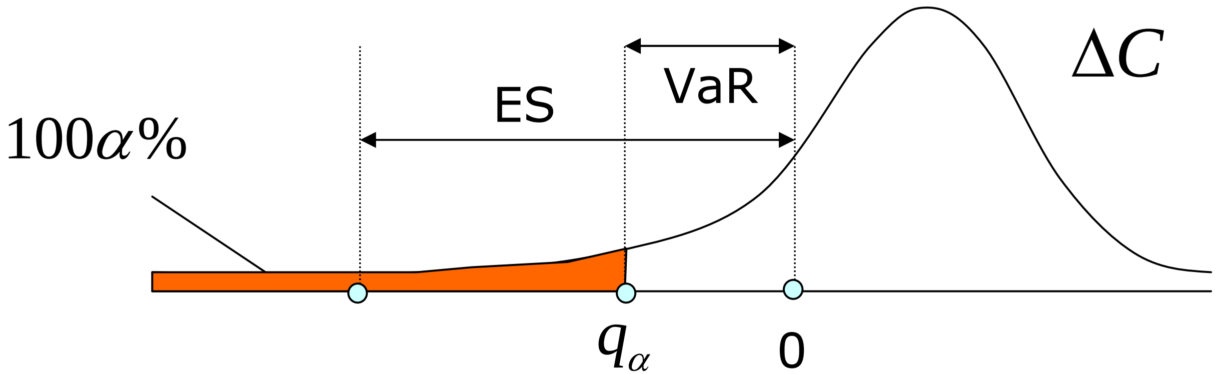
Required Risk Capital (RC)



Add market value margin (MVM)

for entire settlement of insurance portfolio

Expected Shortfall vs. Value at Risk



$$ES(\Delta C) = \frac{1}{\alpha} \cdot \mathrm{E}\left[\left(-\Delta C - VaR(\Delta C)\right)^{+}\right] + VaR(\Delta C)$$

- Expected Shortfall is coherent
- Optimization and capital allocation is possible (limited for VaR)
- Easy to explain: for α=0.01
    - ES="average one-in-a-hundred-years loss"

Market Value Margin: Cost of Capital Approach

What is a good proxy for the Market Value Margin?

Proposals: Quantile and Cost of Capital Approach

Idea: A buyer (or a run-off company) needs to put up regulatory capital during the run-off period of the portfolio of assets and liabilities

$\rightarrow$ a potential buyer needs to be compensated for the cost of having to put up regulatory capital

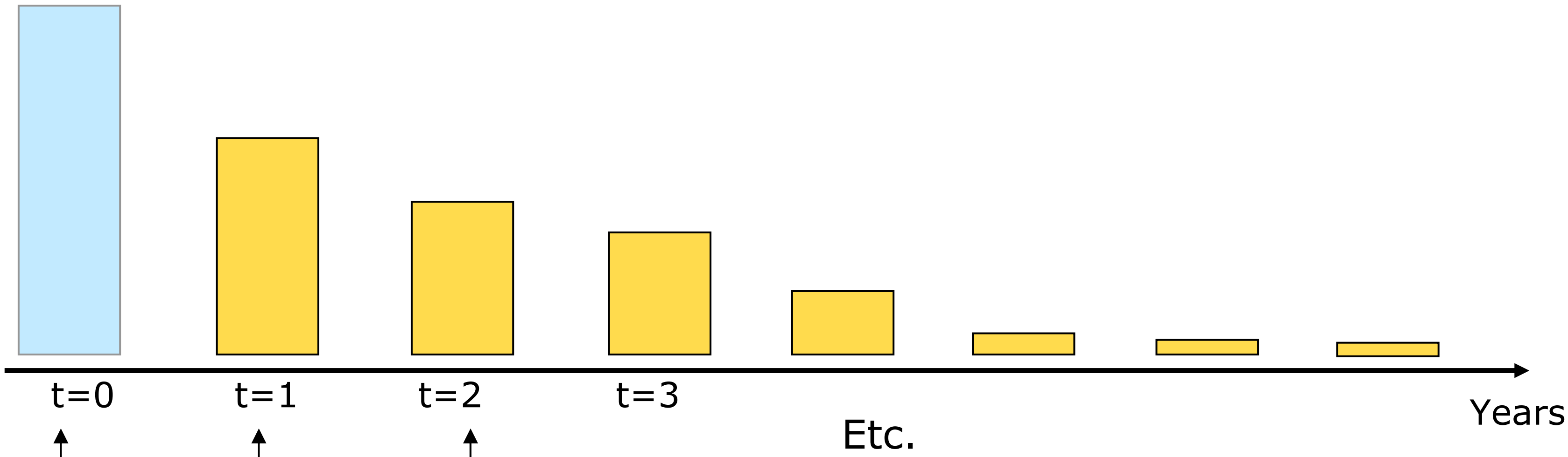
Market Value Margin = the present value of future regulatory risk capital costs associated with the portfolio of assets and liabilities

Problem: How to determine future regulatory capital requirement during the run-off of the portfolio of assets and liabilities?

(by courtesy of Philipp Keller, FOPI, April 2006)

Market Value Margin: Cost of Capital Approach

Risks considered in the MVM:



Risks emanating during year 3 are covered by the cost of setting up ES(ΔC(3)): CoC*ES(ΔC(3)) compensates for having to finance ES(ΔC(3)) during one year

Risks emanating during year 2 are covered by the cost of setting up ES(ΔC(2)): CoC*ES(ΔC(2)) compensates for having to finance ES(ΔC(2)) during one year

Risks emanating during year 1 are covered by ES(ΔC(1))

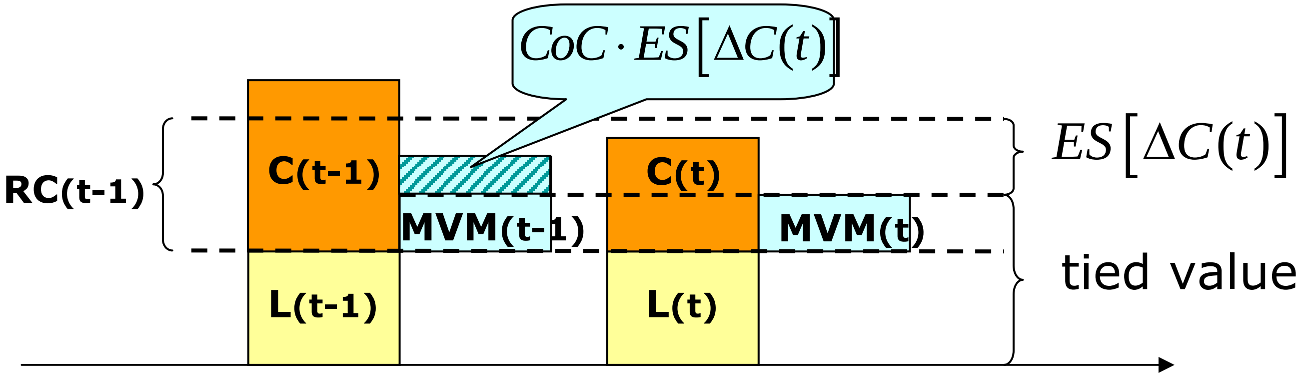
(by courtesy of Philipp Keller, FOPI, April 2006)

Market Value Margin: Cost of Capital Approach

$$RC = ES\left[\Delta C(1)\right] + MVM(1)$$

MVM = cost of capital for future ES

$$MVM(t-1) := CoC \cdot \left(ES\left[\Delta C(t)\right] + \ldots + ES\left[\Delta C(T)\right]\right)$$



Cost of capital spread CoC provided by regulator

Market Value Margin: Cost of Capital Approach

Example:

- Suppose best estimate of liabilities E[L(t-1)] is a good proxy for ES(ΔC(t))
- Determine ES(ΔC(1)) without current year risk (premium risk) and assume assets are optimally replicating liabilities → only unhedgeable financial market risk needs to be considered. Call this value: ES(1)
- Then set **ES(ΔC(t)):=ES(1)/L(0)*E[L(t-1)]**, t=1,...,T

Year t	1	2	3	4	5	6	7
Proxy: Best Estimate of Liabilities	**200**	**150**	**110**	**70**	**40**	**20**	**10**
ES(t) with optimal replicating portfolio	**20,00**	15,00	11,00	7,00	4,00	2,00	1,00
Capital Charge (with 6% cost of capital)	1,20	0,90	0,66	0,42	0,24	0,12	0,06
Discount factor (flat 3% assumed)	1,00	0,97	0,94	0,92	0,89	0,86	0,84
Discount Cost (6%)	1,20	0,87	0,62	0,38	0,21	0,10	0,05
Cost of Capital Margin	**3,45**						

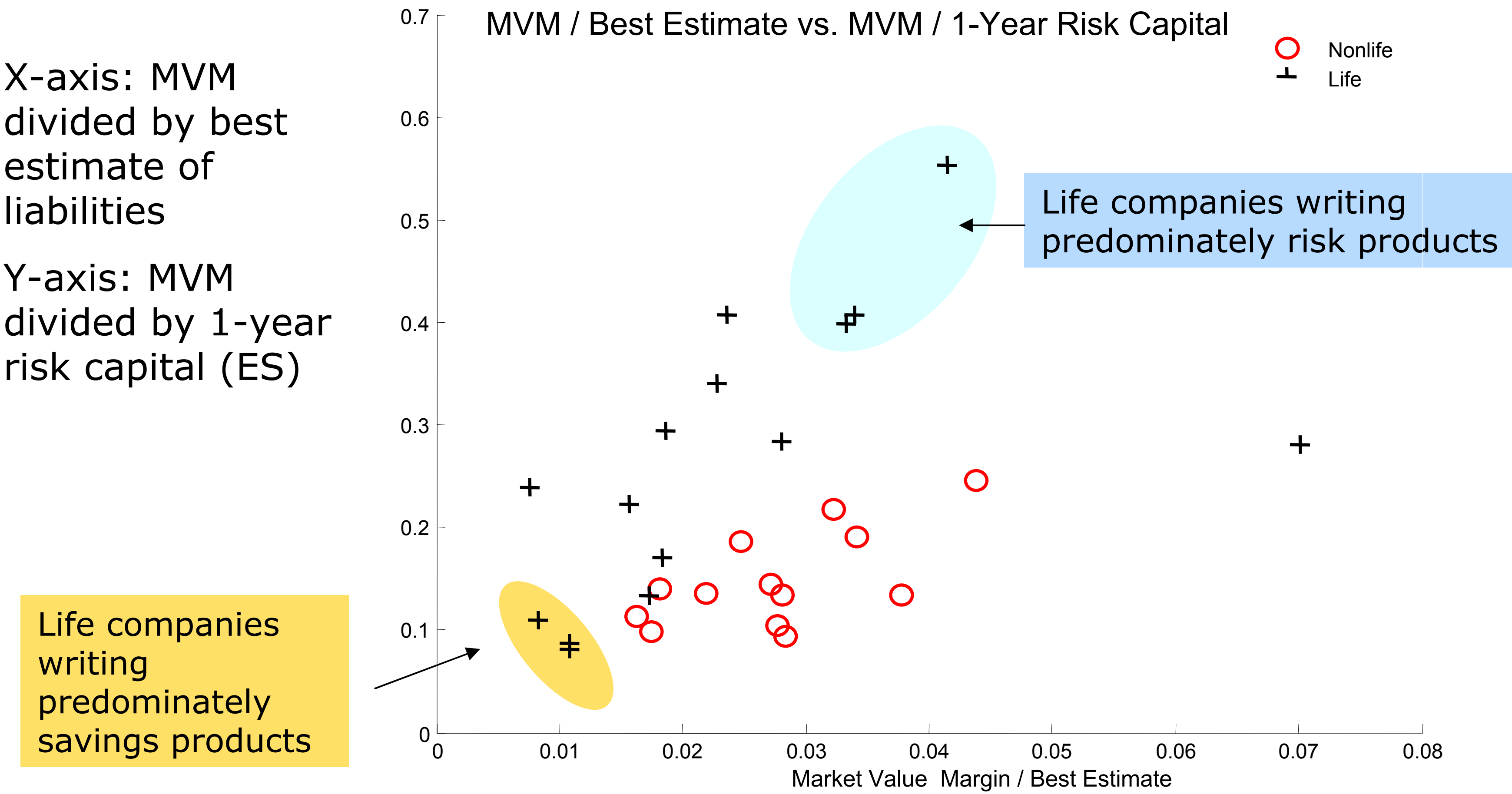
(by courtesy of Philipp Keller, FOPI, April 2006)

Results of the SST Field Tests: MVM

Market Value Margin / Best Estimate vs. Market Value Margin / ES[ΔC], based on provisional data of Field Test 2005

X-axis: MVM divided by best estimate of liabilities

Y-axis: MVM divided by 1-year risk capital (ES)



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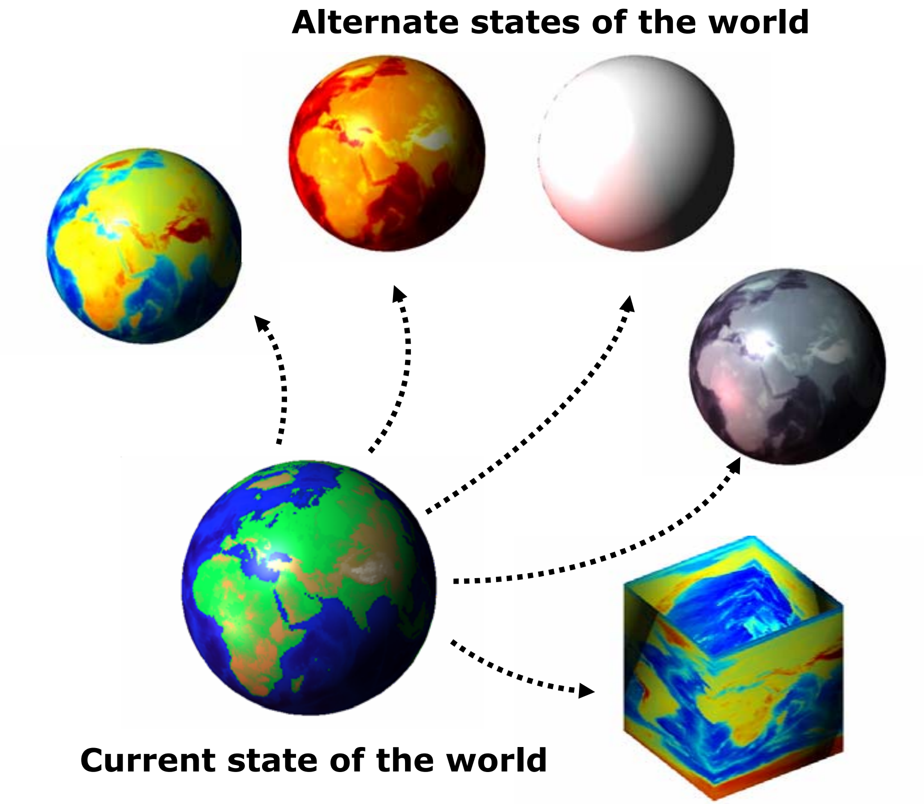
The SST Concept: Scenarios

Scenarios can be seen as thought experiments about possible future states of the world.

Scenarios should illuminate possible but perhaps extreme situations.

Scenarios are different from sensitivity analysis where the impact of a (small) change of a single variable is evaluated.



(by courtesy of Philipp Keller, FOPI, April 2006)

The SST Concept: Scenarios

The formulation and evaluation of the scenario should entail a detailed and comprehensive discussion not only of primary but also of secondary and tertiary effects.

Example: A scenario 'Earthquake in Tokyo' should not only specify the financial impact due to loss of life and to the collapse of buildings, but also discuss the implication on the financial markets (e.g. the collapse of the global financial market for a given duration, the effect on global markets of Japan having to rebuild the infrastructure, etc.).

The formulation of the scenario should comprise

a) the event occurring during the accounting year

b) the effects of the scenario in the future

(by courtesy of Philipp Keller, FOPI, April 2006)

The SST Concept: Scenarios

Historical Scenarios: Stock Market Crash 1987, Nikkei Crash 1989, European Currency Crisis 1992, US Interest Rates 1994, Russia / LTCM 1998, Stock Market Crash 2000

Financial Distress: Increase of i.r., lapse, no new business, downgrading of company,…

Deflation: decrease of i.r.

Pandemic: Flu Pandemic with given parameters (e.g. number of death, sick-days, etc.)

Longevity

Reserving: Provisions have to be increased by 10%

Hail (Swiss specific): Given footprints

Default of Reinsurer: Reinsurer to which most business has been ceded defaults

Industrial Accident: Accident at chemical plant

Personal Accident: large accident during company outing or mass panic in soccer stadium

Anti-selection for Health Insurers: all insured with age < 45 lapse

Collapse of a dam (Swiss specific)

Terrorism

Global Scenarios (for groups&reinsurers)

Property Cats (earthquake, windstorm)

Special Line Cats: Aviation (2 planes collide, marine event, energy event, credit&surety event

(by courtesy of Philipp Keller, FOPI, April 2006)

Modelling Concept for Scenarios

normal year: distribution

Weight (1-p)

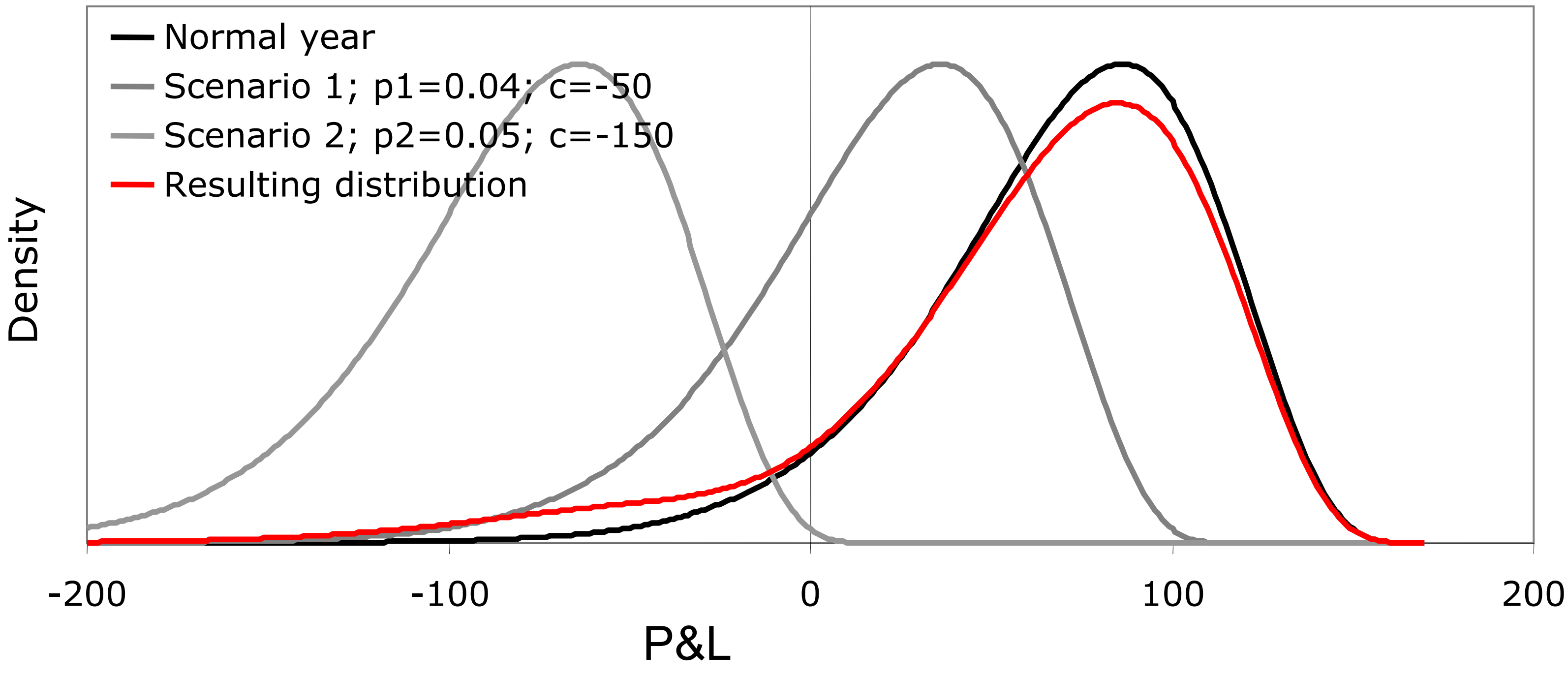
$\Theta = \vartheta_0$

extremal year: scenarios

Weight p

$\Theta = \vartheta_1$

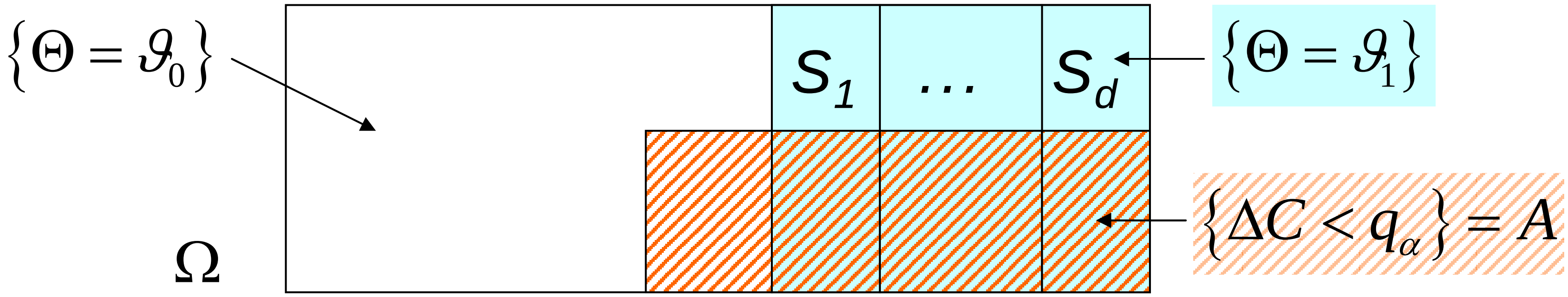
aggregation



Modelling of Scenarios

$$ES\left[\Delta C\right] = \frac{1}{\alpha}\Big((1-p)\cdot \mathrm{E}_{\vartheta_0}\left[-\Delta C \cdot 1_A\right] + p \cdot \mathrm{E}_{\vartheta_1}\left[-\Delta C \cdot 1_A\right]\Big)$$

Extremal year $\left\{\Theta = \vartheta_1\right\} = \bigcup_{i=1}^{d} S_i$ for scenarios $S_1,\ldots,S_d$



$$\mathrm{E}_{\vartheta_1}\left[-\Delta C \cdot 1_A\right] = \frac{1}{p}\sum_{i=1}^{d} \mathrm{E}\left[-\Delta C \cdot 1_A \mid S_i\right] \cdot \mathrm{P}\left[S_i\right]$$

Evaluation of Scenarios

Evaluation of scenarios $\mathrm{E}\left[-\Delta C \cdot 1_A \mid S_i\right]$

Simple Ansatz $\Delta C \mid S_i \sim -c_i + d_i \cdot Y$

where $c_i = \text{expected additional loss amount}$

$d_i = \text{scaling factor}$

$Y \sim \Delta C \mid \Theta = \vartheta_0$: normal year distribution

Example hailstorm: footprints for loss severity; c_i according to market share

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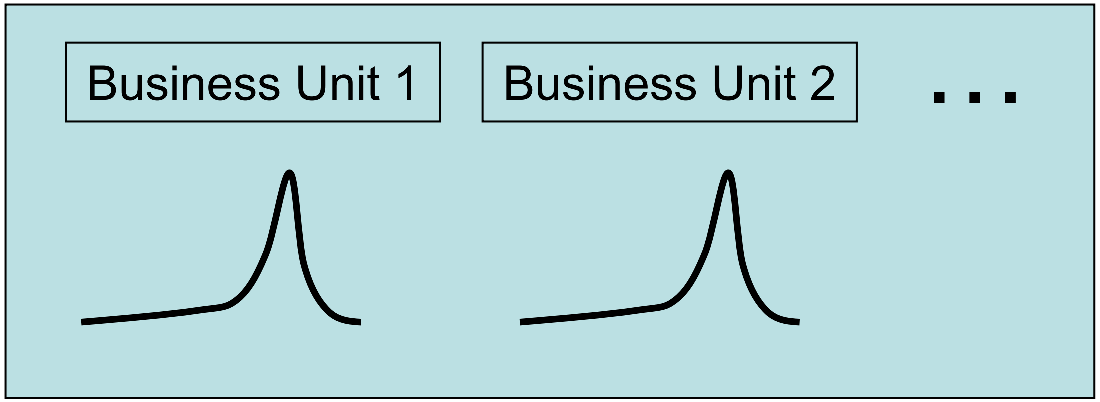
Group Diversification

Diversification is at the core of insurance and other financial business.

Business structure counts!

Insurance Group



Industry Standard: Covariance Method (example)

P&L = sum of centred normal distributed positions:

$$\Delta C = C(1) - C(0) = \underbrace{Z_1 + \ldots + Z_n}_{\text{“Diversification”}}$$

Risk measure linear in standard deviation (VaR, TailVaR):

$$\rho(X) = \kappa \sqrt{Var(X)} + E\left[-X\right]$$

Capital allocation via marginal method:

$$k_i = \frac{d}{dt}\rho\left(\Delta C + tZ_i\right)_{t=0} = \kappa \frac{\sum_j Cov\left(Z_i, Z_j\right)}{\sqrt{Var\left(\Delta C\right)}}$$

Industry Standard: Covariance Method (example)

	Geography A			Geography B			…
Risk type	BU1	BU2	…	BU1	BU2	…	…
Market	k						
Credit							
Non-life Insurance							
Life Insurance							

Level 1: within risk types

$$k = \kappa \sqrt{Var(Z)}$$

(CRO Forum: "A framework for incorporating diversification in the solvency assessment of insurers")

Industry Standard: Covariance Method (example)

	Geography A			Geography B			…
Risk type	BU1	BU2	…	BU1	BU2	…	…
Market	k						
Credit							
Non-life Insurance							
Life Insurance							

<u>Level 2</u>: across risk types within BUs

$$k = \kappa \frac{\sum_{BU1} Cov\left(Z, Z_j\right)}{\sqrt{Var\left(\sum_{BU1} Z_j\right)}}$$

(CRO Forum: "A framework for incorporating diversification in the solvency assessment of insurers")

Industry Standard: Covariance Method (example)

	Geography A			Geography B			…
Risk type	BU1	BU2	…	BU1	BU2	…	…
Market	k						
Credit							
Non-life Insurance							
Life Insurance							

Level 3: across BUs within geography

$$k = \kappa \frac{\sum_{GeographyA} Cov(Z, Z_j)}{\sqrt{Var\left(\sum_{GeographyA} Z_j\right)}}$$

(CRO Forum: "A framework for incorporating diversification in the solvency assessment of insurers")

Industry Standard: Covariance Method (example)

	Geography A			Geography B			…
Risk type	BU1	BU2	…	BU1	BU2	…	…
Market	k						
Credit							
Non-life Insurance							
Life Insurance							

Level 4: across geographies or regulatory jurisdictions

$$k = \kappa \frac{\sum_{Group} Cov\left(Z, Z_j\right)}{\sqrt{Var\left(\sum_{Group} Z_j\right)}}$$

(CRO Forum: "A framework for incorporating diversification in the solvency assessment of insurers")

Fallacy

Individual capital requirement is **not** decreasing with increasing size of the group:

Example:
3 BUs

A		B
BU1	BU2	BU3
k1	k2	k3

$$correlation = \begin{pmatrix} 1 & 0 & 1 \\ & 1 & 0 \\ & & 1 \end{pmatrix}$$

Level 1 (stand alone): $k1 = k2 = k3 = 100$

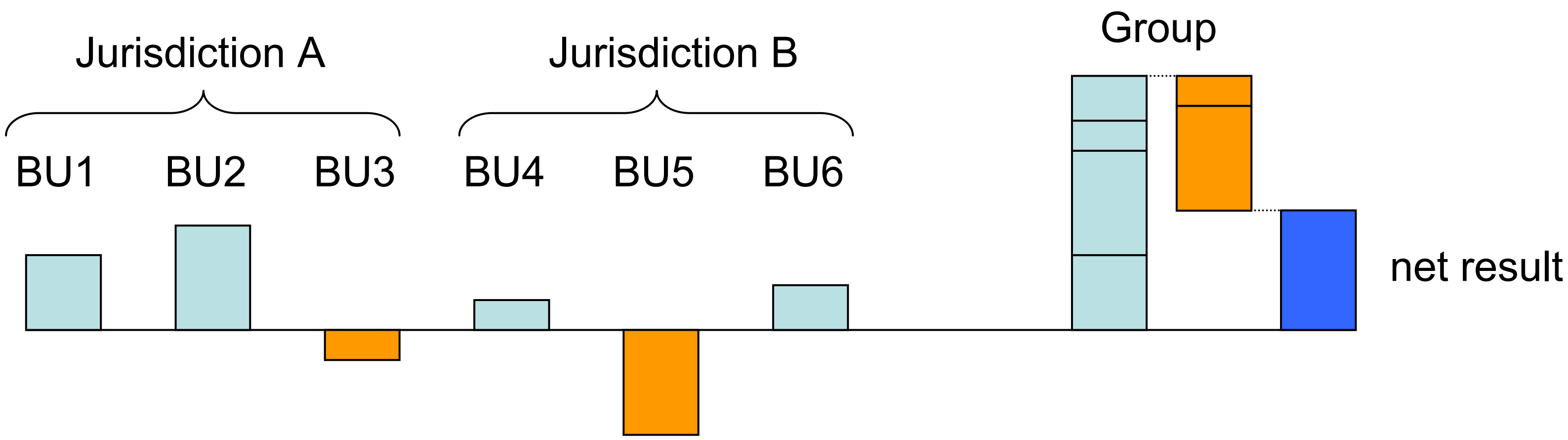
Level 3 (within geography A): $k1 = \frac{1+0}{\sqrt{1+1}} \times 100 = 70.71$

Level 4 (full diversification): $k1 = \frac{1+1}{\sqrt{1+1+1+2\times 1}} \times 100 = 89.44$

→How to define “fairness” for capital allocation methods?

Problem

Diversification effects require full fungibility of capital:



Regulatory risk: regulators may prevent capital to be transferred between jurisdictions

Management risk: companies’ managements may refuse to provide necessary capital injections

→ need of standardization for risk & capital transfers!

Capital and Risk Transfer Instruments

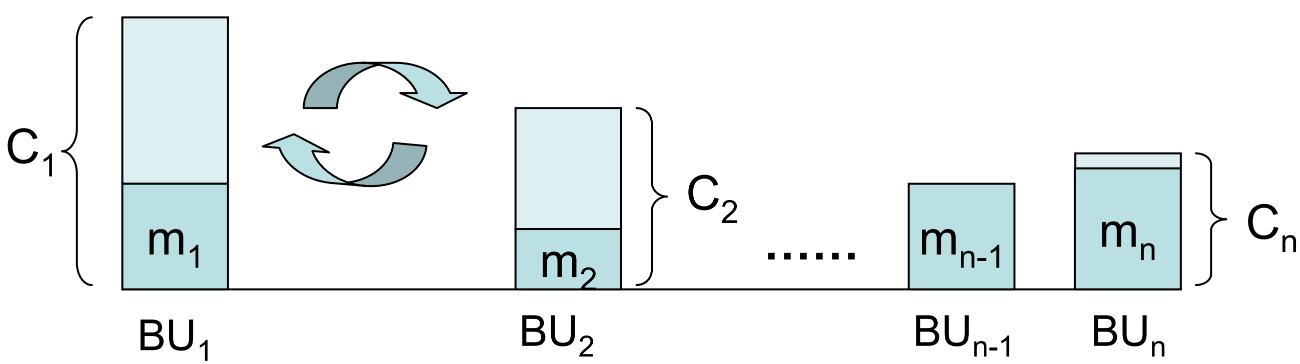
- Intra-group Retrocession
- Guarantees
- Participations
- Dividends
- Loans
- Issuance of Surplus Notes
- Securitization of Future Cash Flows / Earnings
- Etc…

Standardized Diversification. An Example

- *n* BUs with (random) future available capitals $C_1,\ldots,C_n$
- Each BU faces a minimum capital requirement $m_1,\ldots,m_n$ (e.g. RM)
- BUs can share excess gain $(C_i - m_i)^+$ of BU *i* at appropriate price
- New positions of BUs become

$$C_i + \sum_j \lambda_{ij}\left(C_j - m_j\right)^+ + \gamma_i$$

prior position | excess share | cash



Convex risk measures

A **<u>convex risk measure</u>** ρ satisfies

- Monotonicity: $\rho(X) \leq \rho(Y)$ if $X \geq Y$
- Cash invariance: $\rho(X+c) = \rho(X)-c$ for deterministic cash c
- Convexity: $\rho(s \cdot X+(1-s) \cdot Y) \leq s \cdot \rho(X) + (1-s) \cdot \rho(Y)$ for $0 \leq s \leq 1$

ρ is **<u>coherent</u>** if, in addition, $\rho(t \cdot X)=t \cdot \rho(X)$ for positive t

Examples:

- expected shortfall (TailVaR)
- VaR fails in general

Assumption: local regulator specifies the risk measure for every business unit (e.g. SST: expected shortfall, …)

Optimization Problem

(Filipovic, Kupper (2006) „Optimal Capital and Risk Transfers for Group Diversification“)

Find optimal capital and risk transfer (λ,γ) which **minimizes** the group required capital

$$\sum_i \rho\left(C_i + \sum_j \lambda_{ij} (C_j - m_j)^+ + \gamma_i \right)$$

subject to the **clearing condition**

$$\sum_i \left(C_i + \sum_j \lambda_{ij} (C_j - m_j)^+ + \gamma_i \right) \leq \sum_i C_i$$

Question: how does this change the value of the asset-liability portfolio (=available capital)?

Relation to the traditional approach

Assume ρ is coherent, write $C = C_1 + \ldots + C_n$:

$$\rho(C) = \sum_i s_i \cdot \rho(C) = \sum_i \rho(s_i \cdot C) = \min_{\sum X_i = C} \sum_i \rho(X_i)$$

for all convex combinations ($s_1 + ... + s_n = 1$, $s_i \geq 0$)

Convex risk sharing is optimal

Relation to the traditional approach

Assume ρ is coherent, write $C = C_1 + \ldots + C_n$:

$$\rho(C) = \sum_i s_i \cdot \rho(C) = \sum_i \rho(s_i \cdot C) \le \min_{\sum X_i = C} \sum_i \rho(X_i)$$

Choice of X_i limited!

for all convex combinations ($s_1 + \ldots + s_n = 1$, $s_i \geq 0$)

Convex risk sharing is optimal

But not realistic!

our example: $$X_i = C_i + \sum_j \lambda_{ij} (C_j - m_j)^+ + \gamma_i$$

Main result

There exists an optimal capital and risk transfer (λ*,γ*)
if and only if
there exists a linear equilibrium **valuation principle V:**

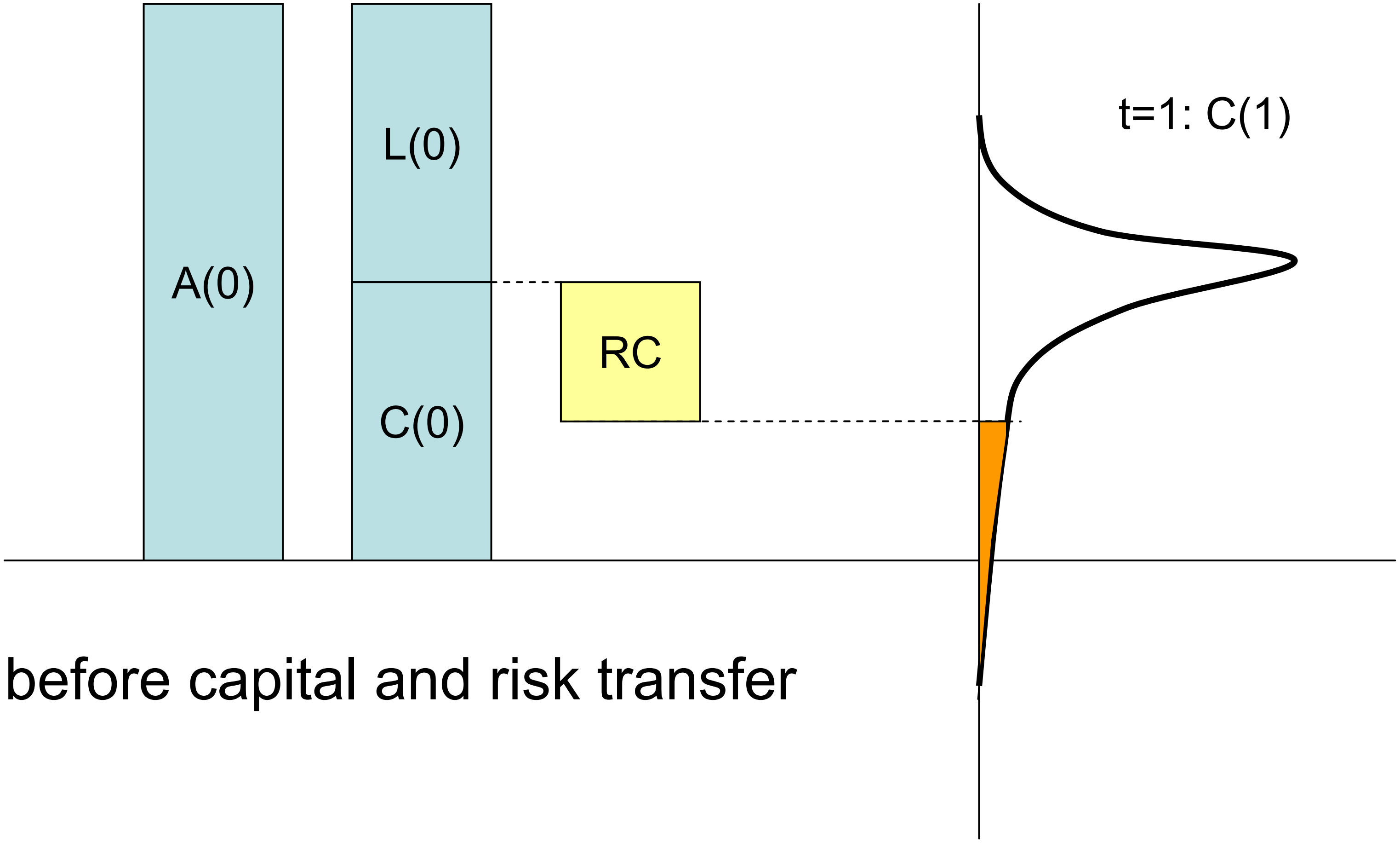
$$\rho\left(C_i + \sum_j \lambda_{ij}^* \left(C_j - m_j\right)^+ + \gamma_i^* \right) = \min \rho\left(C_i + \sum_j \lambda_j \left(C_j - m_j\right)^+ + \gamma \right)$$

$$\text{subject to } V\left(\sum_j \lambda_j \left(C_j - m_j\right)^+ + \gamma \right) = 0$$

“Every business unit achieves maximal reduction of required capital by the C&R transfer (λ*,γ*) among all C&R transfers which leave the available capital invariant.”

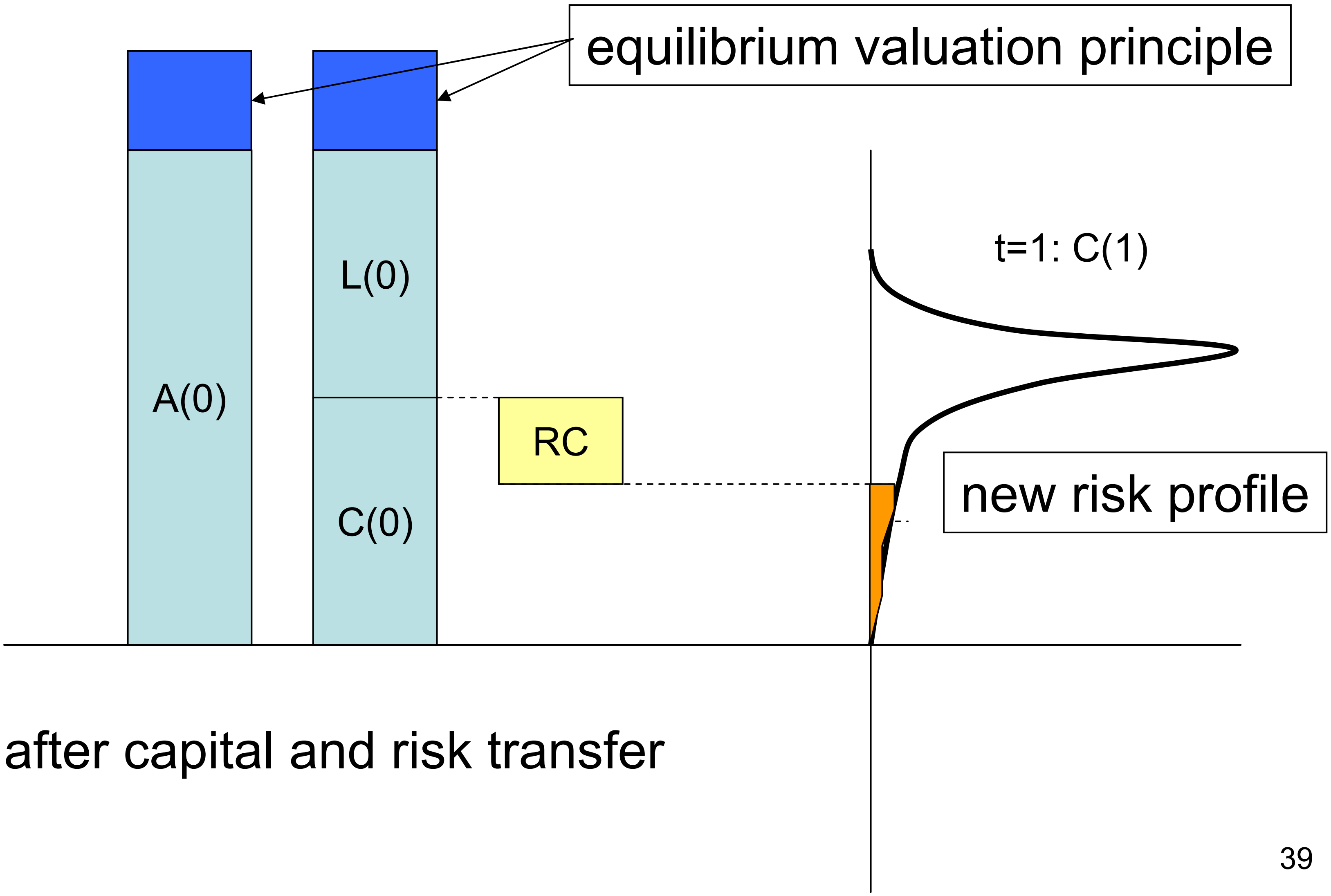
Diversification = reduction of required capital RC

Diversification = reduction of RC



before capital and risk transfer

Diversification = reduction of RC



after capital and risk transfer

Byproduct 1

Fair capital allocation:

$$k_i = \rho\left(C_i + \sum_j \lambda_{ij}^* (C_j - m_j)^+ + \gamma_i^* \right), \quad i = 1,\ldots,n$$

In the sense that:

$$\sum_{i \in I} k_i \leq \min \sum_{i \in I} \rho\left(C_i + \sum_j \lambda_{ij} (C_j - m_j)^+ + \gamma_i \right)$$

for all levels of diversification ("coalitions") $I \subseteq \{1,\ldots,n\}$

→Game Theory

Byproduct 2

Fair linear valuation principle V for risk transfers $(C_i - m_i)+$,
in the sense that:

$$\rho\left(C_i^* + Y\right) \geq \rho\left(C_i^* + V(Y)\right)$$

for every $$Y = \sum_j \lambda_j \left(C_j - m_j\right)^+ + \gamma$$

where $$C_i^* = C_i + \sum_j \lambda_{ij}^* \left(C_j - m_j\right)^+ + \gamma_i^*$$

is the optimal risk profile of business unit i.

→indifference principle

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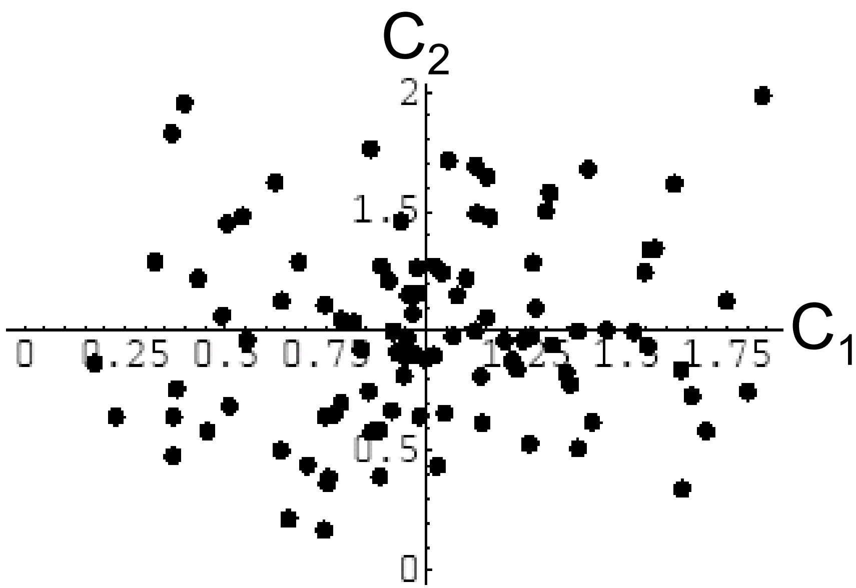
Example: group with two business units

Available capital

- today: $C_1(0) = C_2(0) = 100$ (Mio EUR)
- in one year: 100 states, mean = 100



Risk measure: 99% TailVaR

Example: group with two business units

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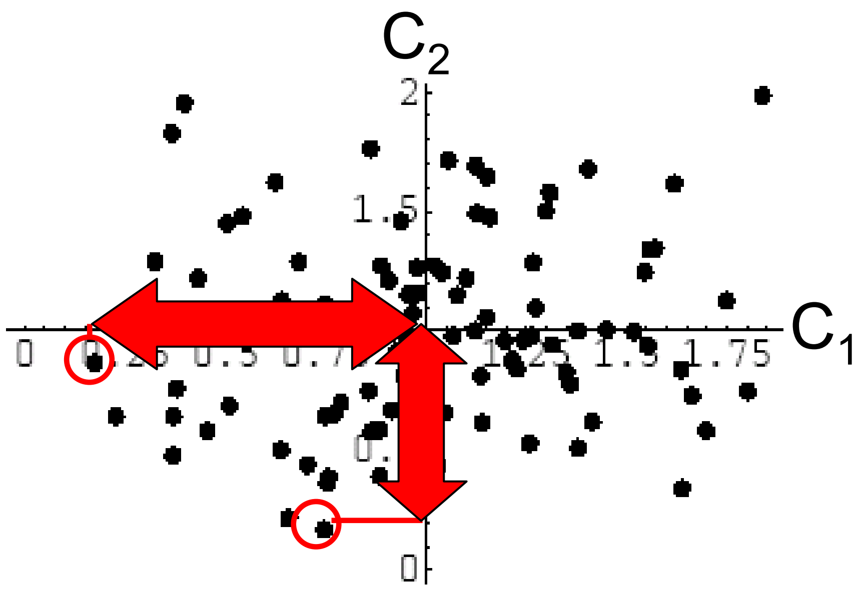


Required capital

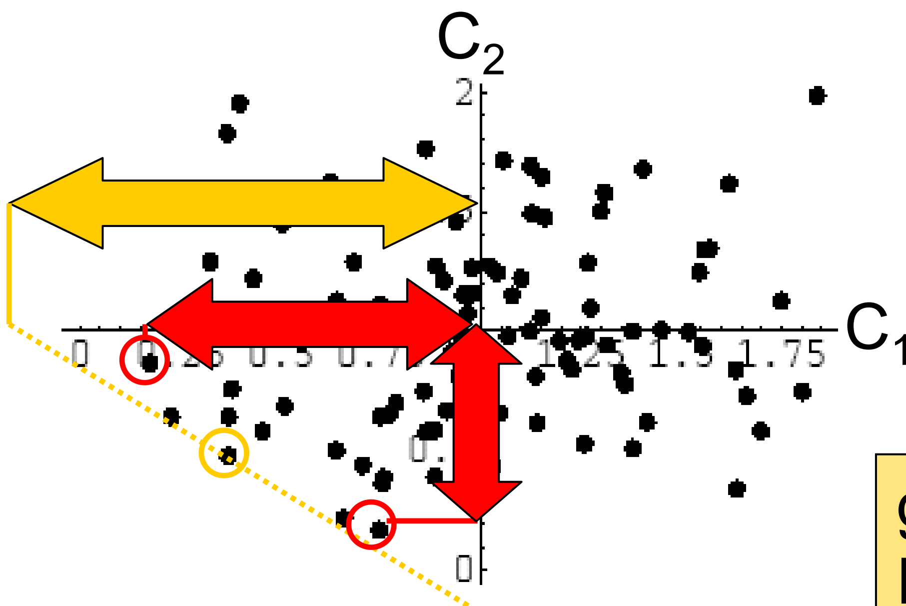
Stand alone
$RC_1= 82.5$
$RC_2= 82.9$

Risk measure: 99% TailVaR

Example: group with two business units

Available capital

- today: $C_1(0) = C_2(0) = 100$ (Mio EUR)
- in one year: 100 states, mean = 100



Risk measure: 99% TailVaR

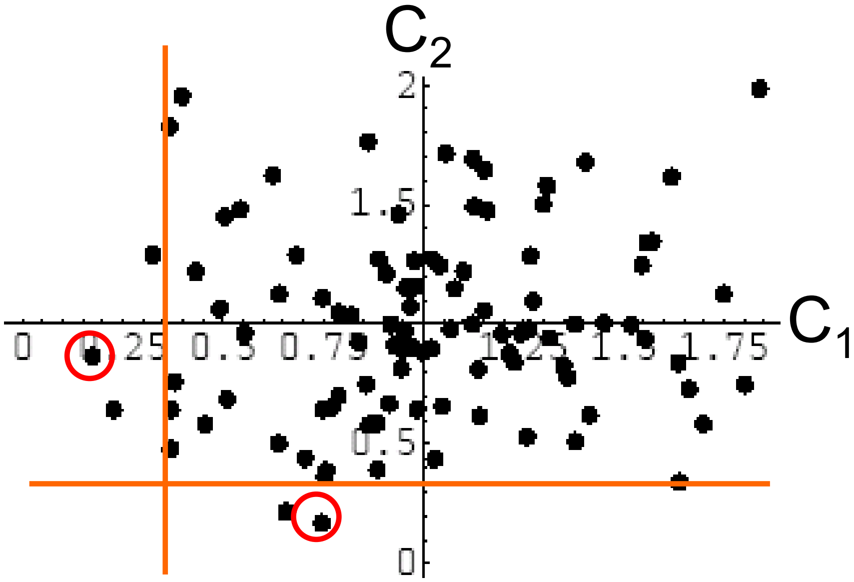
Required capital

Stand alone
$RC_1 = 82.5$
$RC_2 = 82.9$

group full diversified
$RC_{vd} = 115$ ($< RC_1 + RC_2$)

Example: group with two business units

Minimum capital = 40% RC (→SST): m_1=33, m_2=33.2



Example: group with two business units

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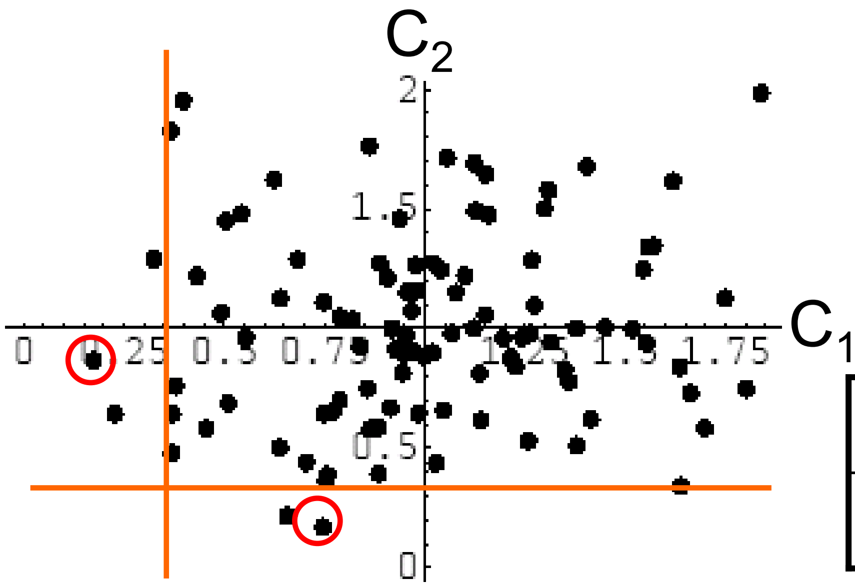


Optimal risk transfer

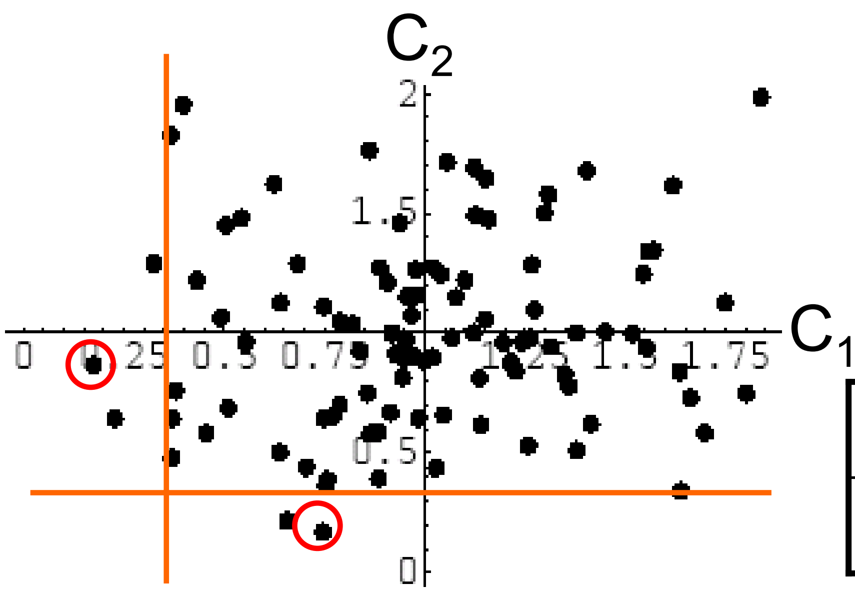
λ_{11}	λ_{12}	λ_{21}	λ_{22}
-0.58	0.75	0.50	-0.81

Equilibrium values

$O_1=(C_1-m_1)^+$	$O_2=(C_2-m_2)^+$
4	15

Example: group with two business units

Minimum capital = 40% RC (→SST): m_1=33, m_2=33.2



Optimal risk transfer

λ_{11}	λ_{12}	λ_{21}	λ_{22}
-0.58	0.75	0.50	-0.81

Equilibrium values

$O_1=(C_1-m_1)^+$	$O_2=(C_2-m_2)^+$
4	15

new positions	RC (prior)
C1 – 0.58 x (O_1 – 4) + 0.75 x (O_2 – 15)	63 (82.5)
C2 + 0.50 x (O_1 – 4) – 0.81 x (O_2 – 15)	52 (82.9)

Example: group with two business units

New positions	RC (prior)
$C_1 - 0.58 \times (O_1 - 4) + 0.75 \times (O_2 - 15)$	63 (82.5)
$C_2 + 0.50 \times (O_1 - 4) - 0.81 \times (O_2 - 15)$	52 (82.9)
sum = $C_1 + C_2 - 0.08 \times O_1 - 0.06 \times O_2 + 1.21$	115

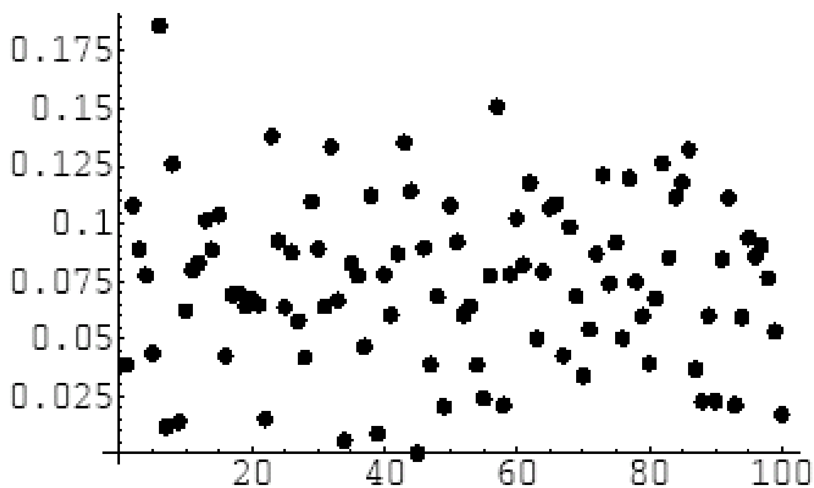
full diversification effect!

Example: group with two business units

New positions	RC (prior)
$C_1 - 0.58 \times (O_1 - 4) + 0.75 \times (O_2 - 15)$	63 (82.5)
$C_2 + 0.50 \times (O_1 - 4) - 0.81 \times (O_2 - 15)$	52 (82.9)
sum = $C_1 + C_2 - 0.08 \times O_1 - 0.06 \times O_2 + 1.21$	115

full diversification effect!

Clearing condition:
sum $\leq C_1 + C_2$



remainder
in 100 Mio EUR
for 100 model points
(→corporate line)

Summary

New risk-based solvency standards for European insurance industry: Solvency II, Swiss Solvency Test

Group diversification effects: restricted capital mobility

→new method for risk aggregation via legally enforceable **capital and risk transfers**

→Available capital remains (equilibrium values) while required capital is minimized (optimization)

→Diversification becomes legally (regulatory) feasible